QMS 110: Linear Inequalities

TRAIN TO LEARN EFFECTIVELY: TIP SHEETS

CONTENT BY: LAUREN RODRIGUEZ

WHAT ARE LINEAR INEQUALITIES?

A linear inequality is a linear function that is expressed as a relation between two numbers or variables that are not always equal to each other.

PROPERTIES OF LINEAR INEQUALITIES:

1. If $a < b$, then $a + c < b + c$ and $a - c < b - c$
2. a) If $a < b$ and c is positive (+), then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$

   b) If $a < b$ and c is negative (-), then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$
3. Transitive Property: If $a < b$ and $b < c$, then $a < c$

ABSOLUTE INEQUALITIES:

If $a > 0$, then

1. $|u| < a$, if and only if $-a < u < a$
2. $|u| > a$, if and only if $u < -a$ OR $u > a$

SYSTEM OF LINEAR INEQUALITIES:

What is a System of Linear Inequalities?

Two or more linear inequalities that are viewed together as a unit.

How do you solve a System of Linear Inequalities?

On the graph of a system of linear inequalities, the overlapping region that contains the solution to both inequalities is the solution of the entire system.

Steps:

1. Graph the system of linear inequalities
   a) Find the x and y intercepts of all the linear inequalities in the system

      TIP: When trying to graph a linear inequality, treat the linear inequality as if it is a linear equation **Ex:** $2x - y \leq 6$ ➡ $2x - y = 6$

      NOTE: To graph a linear equation, since only two points on the line are needed to be able to graph, we are finding the x and y intercepts as they are the easiest to determine.

      REMEMBER: To find the x-intercept, set y = 0 | To find the y-intercept, set x = 0

   b) Plot the x and y intercepts and draw the lines of each linear inequality
2. Find the region/which side on the graphed linear inequality line contains the solution/satisfies the linear inequality
   a) Set the variables of each linear inequality = 0 (*this will likely simplify the side of the inequality containing the variables = 0*)

      EX: $2x - y \leq 6$ ➡ $2(0) - (0) \leq 6$ ➡ $0 \leq 6$

Toronto Metropolitan University

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Ted Rogers School of Management, TRS 1-004
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b) See if the inequality is satisfied/correct (**ASK:** *Does this inequality seem accurate?*)
EX - SATISFIED: $0 \leq 6$ | NOT SATISFIED: $0 \geq 2$

c) If the linear inequality is satisfied, the area containing (0,0) has the solution
If the linear inequality is **not** satisfied, the area **not** containing (0,0) has the solution

3. The area/region that contains the solution for the system of linear inequalities is the solution area

LINEAR PROGRAMMING:

What is Linear Programming?

Linear Programming is a specific type of system of linear inequalities, where the goal is to either maximize or minimize a given equation (*typically a profit or cost equation*).

How do you solve a Linear Programming question?

Linear Programming uses the same general method as solving for a system of linear inequalities, however, it contains a few additional steps.

Steps:

1. Solve the system of linear inequalities (*refer to the steps above*)
2. Find the boundary points of the solution area
   - Oftentimes, the boundary points (may) include the origin, an x or y intercept, or the point of intersection (POI) of two linear inequalities
   - When determining the POI: treat the intersecting linear inequalities as a system of linear equations, and solve for that system (refer to our Solving a System of Linear Equations Tip Sheet for more detail)
3. For each of the boundary points, plug the x and y coordinates into the x and y values of the given equation that is to be maximized or minimized, and solve for a single value
4. **Maximizing:** The optimal point is the one that provides the **highest** singular value to the equation out of all the possible boundary points
   Minimizing: The optimal point is the one that provides the **lowest** singular value to the equation out of all the possible boundary points

LINEAR PROGRAMMING EXAMPLE:

A company is looking to maximize their profit. The company's profit equation is: $P(x) = 5x + 5y$. The maximization of their profit is subject to the following:

(1) $2x + y \leq 10$
(2) $x + 2y \leq 8$
(3) $x, y \geq 0$

What set of x and y values should the company use to maximize their profit?

Graph the system of linear inequalities.

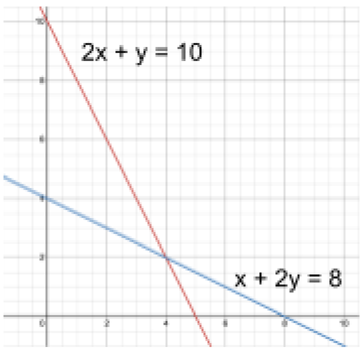
(1) X-intercept: $2x + (0) = 10$ ➡ $2x = 10$ ➡ $x = \frac{10}{2}$ ➡ $x = 5$ | x-intercept = (5,0)

Y-intercept: $2(0) + y = 10$ ➡ $y = 10$ | y-intercept = (0,10)

(2) X-intercept: $x + 2(0) = 8$ ➡ $x = 8$ | x-intercept = (8,0)

Y-intercept: $(0) + 2y = 8$ ➡ $2y = 8$ ➡ $y = \frac{8}{2}$ ➡ $y = 4$ | y-intercept = (0,4)

NOTE: $x, y \geq 0$ indicates that the solution area of the system contains only positive values/is only bounded to the first quadrant.

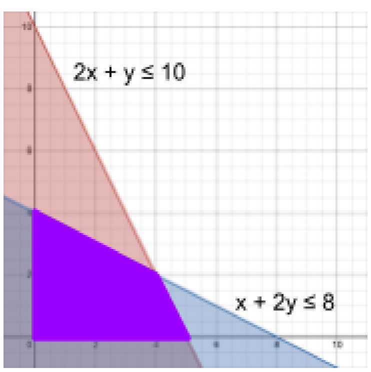


Key: Red - (1) | Blue - (2)

Find the region/which side on the graphed linear inequality line contains the solution/satisfies the linear inequality.

(1) $2(0) + (0) \leq 10$ ➡ $0 \leq 10$, satisfied (the side containing the origin is the solution area)

(2) $(0) + 2(0) \leq 8$ ➡ $0 \leq 8$, satisfied (the side containing the origin is the solution area)



Find the boundary points of the solution area.

POI using elimination:

1. After choosing the variable to eliminate (in this example, the variable y), take the two linear equations (*we will treat our given linear inequalities as equations for the sake of finding our POI*), and determine how to multiply the equation(s) so the variable to be eliminated has coefficients that when added will equal zero (*typically the coefficients will have the same absolute value, with one being negative and the other positive*)

NOTE: In some cases, at least one of the original linear equations does not need to be multiplied. The key factor is making sure to manipulate the linear equations so the coefficients of the desired variable can be eliminated when added.

NOTE: When multiplying a linear equation to set it up for elimination, it is important to multiply **every element** of the equation by the same number, as to not change the value of the equation.

In this example, we will be multiplying (1) by -2, and keeping (2) in its original form.

(1) $2x + y = 10$ ➡ $-4x - 2y = -20$

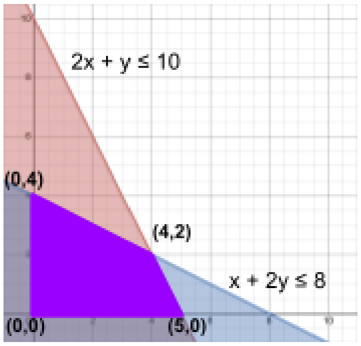
(2) $x + 2y = 8$ ➡ $x + 2y = 8$

2. Add both of the equations (*if done correctly, there should only be one variable remaining*). Isolate and solve for the remaining (not eliminated) variable.

$-3x = -12$ ➡ $x = \frac{-12}{-3}$ ➡ $x = 4$

3. Substitute the found value into one of the original linear equations and solve for the second variable. In this example, we will substitute our found value of 4 in for the x variable in (1).

(1) $2(4) + y = 10$ ➡ $8 + y = 10$ ➡ $y = 10 - 8$ ➡ $y = 2$ **POI:** $(4, 2)$



Find the optimal point to maximize the company's profit equation.

$P(x) = 5(0) + 5(0) = 0$

$P(x) = 5(0) + 5(4) = 20$

$P(x) = 5(4) + 5(2) = 30$ → This is the optimal point to maximize the company's profit

$P(x) = 5(5) + 5(0) = 25$