ITM 107

Sets, Linear Equations and Functions, Quadratics

Sets:

Definitions:

- **Set:** a collection of objects
- **Element/Member of a Set:** an object in a set
- **Finite Set:** elements can be listed
  - Example: A={1,2,3,4,5}
- **Infinite Set:** elements can't be listed
  - Example: N = {1,2,3,4,5,....}
- **Empty/ Null Set:** a set with no elements – written as ∅ or {}
- **Universal Set (U):** a set which contains all other sets under the discussion
- **Representing sets:**
  - Listing: A = {1,2,3,4,5}
  - Description:. A= {x: x∈N and x<6}

Relations:

- **Equal:** A=B when they both have the same elements
  - Example: A={1,2,4,5}, B={4,5,2,1} => A=B
- **Subset (⊆):** A is a subset of B (A⊆B) when every element of A is an element of B
  - Example: A={1,2}, B={1,2,3,4} => A⊆B
  - Each set is a subset of itself – Example: **A⊆A**
  - The empty set is a subset of every set – Example: **∅⊆A**
- **Disjoint:** when both sets have no elements in common
  - Example: A={1,2,3}, B = {4,5,6}

Operations:

Given sets A and B

- **Intersection (∩):** A set is the intersection of sets A and B (A∩B) when it contains the **common elements** between A and B
  - Example: A={2,4,6}, B= {2,3,5} => A∩B ={2}

- **Union (∪):** A set is the union of sets A and B (A∪B) when it contains **ALL elements** of sets A and B
  - Example: A={2,4,6}, B= {2,3,5} => A∪B={2,3,4,5,6}
- **Complement ('):** Set A' is a complement set of set A, where it contains all elements in the universal set that **ARE NOT contained in set A.**
  - A'={x: x∈ U and x∉A}
  - Example: U={x: 0<x<9}, A={1,2,3,4} => A'={5,6,7,8}

Linear Equations and Functions:

Functions:

Definitions

- **Relation:** a connection between the elements of 2 sets – represented as an ordered pair (x,y)
- **Function: a relation with a rule** that assigns each element in its domain exactly one element in the range
- **f(x):** f of x is the value of a function f at an element x of its domain.
- **y=f(x):**
  - **x: domain** – considered as input. x is an **independent variable** where you can control the input
  - **y: range** – considered as output. y is a **dependent variable** because it depends on the rule and the input.
  - Example: given function y = f(x) = 4x. As we can select any number for x (independent), there is only one output for y (dependent), which is equal to 4 times x. If we select x=2, then y can only be 8.

Operations:

- **Sum:** $(f + g)(x) = f(x) + g(x)$
- **Difference:** $(f - g)(x) = f(x) - g(x)$
- **Product**: $(f \,.\, g)(x) = f(x).\, g(x)$
- **Quotient:** $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}\ if\ g(x)\ 0$
- **Composition:** $(g\circ f)(x) = g(f(x))$ – replacing the x from g(x) function with f(x) function

- **IMPORTANT: Understand the difference between product and composition**
  - Example: Given f(x) = 2x-1 and g(x) = 3x

    Product: $(f\,.\,g)(x) = f(x).g(x) = (2x - 1)(3x) = 6x^2 - 3x$
    Composition: $(f \circ g)(x) = f(g(x)) = 2(3x) - 1 = 6x - 1$

Linear Functions:

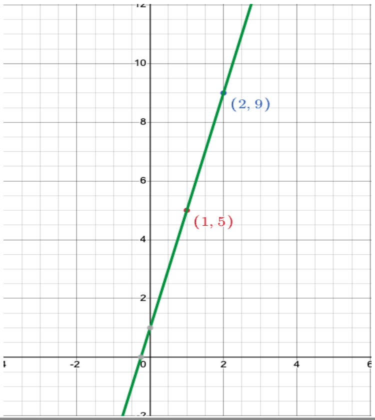
Definitions:

- Form of function: $y = f(x) = ax + b$ where a and b are constants
- Intercepts:
  - **x-intercept:** y = 0. The point where the line intersects the x-axis. The value of the coordinate at that point is (x,0)
  - **y-intercept:** x = 0. The point where the line intersects the y-axis. The value of the coordinate at that point is (0,y)

How to graph a linear function: need at least 2 points

- Find the intercepts and connect those points with a straight line
  OR
- With x as an independent input and y as a dependent input:
  - Step 1: Select any x (small x for easy calculation)
  - Step 2: Plug selected x in the function to find y
  - Step 3: Repeat steps 1 and 2 to find the second point
  - Step 4: Connect the points with a straight line

Example: $y = 4x + 1$

1. Select $x = 1$
2. $y = 4(1) + 1 = 5 \Rightarrow$ Found 1st coordinate (1,5)
3. Select $x = 2 \Rightarrow y = 4(2) + 1 = 9 \Rightarrow$ Found 2nd coordinate (2,9)
4. 

Slopes:

- Slope is the rate of change of a linear function
- Slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

Given m1 is the slope of line 1 and m2 is the slope of line 2

- Parallel lines: **m1 = m2**
- Perpendicular lines: $m2 = \frac{-1}{m1}$ given m1 ≠ 0
- m > 0: function is increasing
- m < 0: function is decreasing
- m = 0: horizontal line, function is constant
- m undefined ($\Delta x = 0$): vertical line, function is undefined

Equations of lines:

General form: $ax + by + c = 0$
Point-slope form: $y - y_1 = m(x - x_1)$
Slope-intercept form: $y = mx + b$
Vertical line: $x = a$
Horizontal line: $y = b$

Example on using the equation of lines:

Given a linear function: $3x - 2y = 3$. Find the slope of this function..

<u>Solution</u>: From the equation of lines, the slope-intercept form helps you find the slope (m).

$$3x - 2y = 3$$

$$\Rightarrow -2y = 3 - 3x$$

$$\Rightarrow y = \frac{-3x+3}{-2}$$

$$\Rightarrow y = \frac{3}{2}x - \frac{3}{2}$$

$$\Rightarrow m = \frac{3}{2}$$

Example 2: Find the equation of a line with slope m = 2 that passes through the point (3,4).

Solution: From the equation of lines, the point-slope form helps you find the equation. Step-by-step solution as below:

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 4 = 2(x - 3)$

$\Rightarrow y - 4 = 2x - 6$

$\Rightarrow y = 2x - 2$ (final equation)

Or you can also write it under the general form: $2x - y - 2 = 0$

Quadratics:

Definitions:

- **Quadratic equation:** the highest power of the variable x is 2
- **How to solve:**
  - Factoring – if simple
  - Quadratic formula: if $ax^2 + bx + c = 0$ *where* $a \neq 0$

    Then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant – the number of solutions:

- $b^2 - 4ac > 0$, two distinct real solutions
- $b^2 - 4ac = 0$, one real solution – vertex point
- $b^2 - 4ac < 0$, no real solution

Vertex point (x,y):

$x = \frac{-b}{2a}$ and $y = f(x) = f(\frac{-b}{2a})$

- a > 0: vertex point is the minimum point
- a < 0: vertex point is the maximum point

Rate of change:

$m = \frac{f(b) - f(a)}{b - a}$

Business application using quadratics:

Break-even point:

- Put the revenue function and the cost function equal to each other => equation
- Turn your new equation into a general form of quadratics $ax^2 + bx + c = 0$
- Solve for x (**NOTE**: In real life, negative numbers don't exist when it comes to production, so be careful when selecting x results)

Max revenue:

- Find the vertex point x, then use x in the revenue function to find the maximum revenue

Max profit:

- Find the vertex point x, then use vertex point x in the formula:
  Profit = Revenue – Cost (revenue function – cost function) if looking for max profit

  Revenue function if looking for max revenue

Market Equilibrium: Demand = Supply:

- q stands for **QUANTITY** and p stands for **PRICE**. Do not mix them up when you solve the equation.
- Solve for q or p (**NOTE**: In real life, negative numbers don't exist when it comes to production, so be careful when selecting your results)