QMS 130: Introduction to Logarithms

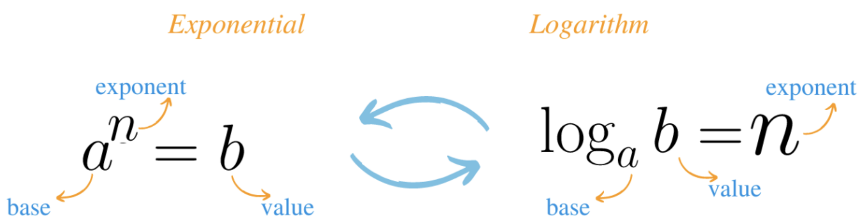
Why Do We Need Logarithms?

Logarithms are the inverse of exponents. Logarithms are used to find the power of a certain exponential function. Let's have a look at this simple example.

$3^x = 81$ (this is an exponential function)

It can be difficult to solve for x. This is where we need logarithms. By changing the above equation into logarithmic form using the formula below, we can solve for x.

$X = \text{Log}_3\ 81$



source: https://www.storyofmathematics.com/logarithm-rules/

Key Terms

"Log 5 to the base 2"	This logarithm is expressed as **$\text{Log}_2\ 5$.**
Natural Logarithm	This is a log to the base of *e (a constant, irrational number which is equal to 2.7183) which is expressed as* **Ln or Log_e**
Common logarithm	These are any logs with a base of 10 for example **$\text{Log}_{10}\ 81$.** These can also be expressed simply as **Log 81.**
Log 1 = 0	The logarithm of 1 is always equal to zero, regardless of the base.
$\text{Log}_2\ 2$	When the logarithm has the same base as the value, it is always equal to 1.

QMS 130: Introduction to Logarithms

Basic Logarithm Rules

Here are some basic logarithmic rules you'll need to solve problems. Let's take a deep look into each of the five rules.

Logarithmic Properties	
Product Rule	$\log_a(xy) = \log_a x + \log_a y$
Quotient Rule	$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
Power Rule	$\log_a x^p = p\log_a x$
Change of Base Rule	$\log_a x = \frac{\log_b x}{\log_b a}$

Product Rule

The product rule says that the logarithm of a product is equal to the sum of logs. This rule helps to break down complex logs into multiple terms.

Remember, that $\log_b(x + y) \neq \log_b x + \log_b y$.

Assume, the question asks you to expand the following logarithmic expression. Here's how you would use the product rule to find the solution.

$Log_3\ 10 = Log_3\ (2 \times 5)$
$Log_3\ 10 = Log_3\ 2 + Log_3\ 5$

Quotient Rule

The quotient rule says that the logarithm of a quotient is equal to the difference of logs.

QMS 130: Introduction to Logarithms

Here's an example

$Log_3 \frac{81}{27}$ = Log_3 81 - Log_3 27

= Log_3 3

= 1

Therefore, $Log_3 \frac{81}{27} = 1$

Power Rule

The power rule can be used to simplify the logarithm of a power by rewriting it as the product of the exponent times the logarithm of the base.

Here's an example of changing a logarithm using the power rule.

$Log_2 10^5 = 5 Log_2 10$

Change of Base Rule

Most of our calculators can only evaluate common logs and natural logs. To solve logarithms with a base other than 10, we need to use the change of base formula to rewrite the logarithm as a quotient of logarithms with any other base.

We usually change the logarithm into a natural log, **log with base 3 (or ln),** or into a common log, **log with base 10 (or just log).**

Here's an example where we need to change the log into a quotient of natural logarithms.

1. $Log_2 10 = \frac{ln\ 10}{ln\ 2}$ = 3.3219 *(using a calculator)*
2. $Log\ 9 = \frac{ln\ 9}{ln\ 10}$ *(remember, "log" means* log_{10}*)*

Program Advising and Student Success
Ted Rogers School of Management, TRS 1-004
https://www.torontomu.ca/trsm-pass/