QMS210: 1s HYPOTHESIS TESTING

TRAIN TO LEARN EFFECTIVELY: TIP SHEETS

Prepared by Greg (2024)

Hypothesis Testing:

The process of hypothesis testing starts with the evaluation of a certain assertion or proposition, with the aim to determine whether it can be rejected or not rejected.

Null Hypothesis: H_0

- The null hypothesis, often denoted as H_0, is the claim that there is no effect or relationship between the variables being studied. In other words, it is the hypothesis that there is no significant difference between specified populations, and any observed difference is due to sampling or experimental error.

- For example, if you are studying the effect of a drug on patient recovery times, the null hypothesis might be that the drug has no effect on recovery times.

- This hypothesis is assumed to be true until the evidence suggests otherwise. If the data collected provides strong enough evidence against the null hypothesis, it is then rejected in favor of an alternative hypothesis.

Alternative Hypothesis (What we input into the calculator): H_a or H_1

- The alternative hypothesis, often denoted as H_a or H_1, is a statement that contradicts the null hypothesis in statistical testing. It proposes that there is a significant effect or relationship between the variables being studied.

- For instance, if you're studying the impact of a drug on patient recovery times, the alternative hypothesis might be that the drug does have a significant effect on recovery times. This hypothesis is considered to be true if the collected data provides strong enough evidence against the null hypothesis.

- The alternative hypothesis is the claim that is tested when attempting to disprove the null hypothesis.

Determining the type of test:

- **T-Test:** Use this when the population standard deviation (σ) is **unknown** and the sample size is small (typically less than 30).

- **Z-Test:** Use this when the population standard deviation (σ) is **known**, regardless of the sample size.

*Note: All Calculator Instructions In This Document Are For Casio fx-9750GII

- **Proportion-Test:** Use this when you are comparing proportions (e.g., the proportion of success in two different groups).
    - Proportion Tests have a binary outcome meaning there are only two possible results; similar to flipping a coin where the outcome can either be heads or tails.
    - To ensure that a sample size is large enough to assume the sampling distribution of the proportion is approximately normally distributed, both of the following conditions should be met:
        1. $n * \pi > 5$
        2. $n * (1 - \pi) > 5$
    - Where (n) is the sample size and (π) is the population proportion. If either condition is less than the number **five** the sample size is too small to assume normality and we cannot conduct the hypothesis test.

Determining the Test: Lower-Tailed, Upper-Tailed, or Two-Tailed Test

- **Lower-Tailed Test:** Use this when you are testing if a parameter is less than a certain value (e.g., testing if the mean is less than a specific value).
- **Upper-Tailed Test:** Use this when you are testing if a parameter is greater than a certain value (e.g., testing if the mean is greater than a specific value).
- **Two-Tailed Test:** Use this when you are testing if a parameter is different from a certain value (e.g., testing if the mean is different from a specific value, either higher or lower).

	Lower-Tail	Upper-Tail	Two-Tail
T-Test	$H_0: \mu \geq \mu_0$ $H_1: \mu < \mu_0$	$H_0: \mu \leq \mu_0$ $H_1: \mu > \mu_0$	$H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$
Z-Test	$H_0: \mu \geq \mu_0$ $H_1: \mu < \mu_0$	$H_0: \mu \leq \mu_0$ $H_1: \mu > \mu_0$	$H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$
Proportion-Test	$H_0: \pi \geq \pi_0$ $H_1: \pi < \pi_0$	$H_0: \pi \leq \pi_0$ $H_1: \pi > \pi_0$	$H_0: \pi = \pi_0$ $H_1: \pi \neq \pi_0$

Where μ_0 and π_0 are the hypothesized averages

Critical Values:

- Critical values are key points on the distribution of a test statistic under the null hypothesis. They define a set of values that call for rejecting the null hypothesis. These values are used to determine whether the results of a statistical test are statistically significant.

- In hypothesis testing, the critical value is a cut-off value that marks the start of a region in which the test statistic is unlikely to fall.

- If the value of the test statistic is less extreme than the critical value, then the null hypothesis cannot be rejected. However, if the test statistic is more extreme than the critical value, the null hypothesis is rejected and the alternative hypothesis is not rejected.

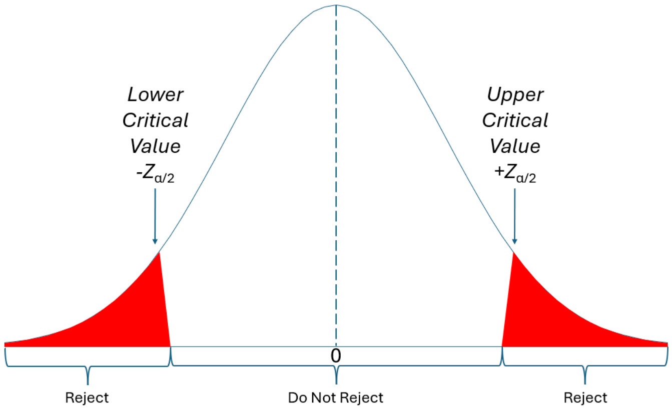
- Critical values depend on your significance level and whether you're performing a one or two-sided hypothesis. For example, for a significance level of 0.05, the critical values define the minimum distance from the null hypothesis required for statistical significance.

When finding critical values for Z and proportion distributions standard deviation (σ) is always equal to **1** and mu (μ) is always equal to **0**. The following examples assume a 0.05 level of significance. Proportion critical values follow the same steps as Z critical values.

One Sample Z Two-Tail Critical Values:

H_0: $\mu = \mu_0$

H_1: $\mu \neq \mu_0$



<u>Calculator Instructions:</u>

From the main menu select:

STAT > **F5**(DIST) > **F1**(NORM) > **F3**(InvN)

<u>Enter the following in Inverse Normal:</u>

Data: **F2** (Var)

Tail: **CNTR**

Area: **0.95** (1 - level of significance)

σ: **1**

μ: **0**

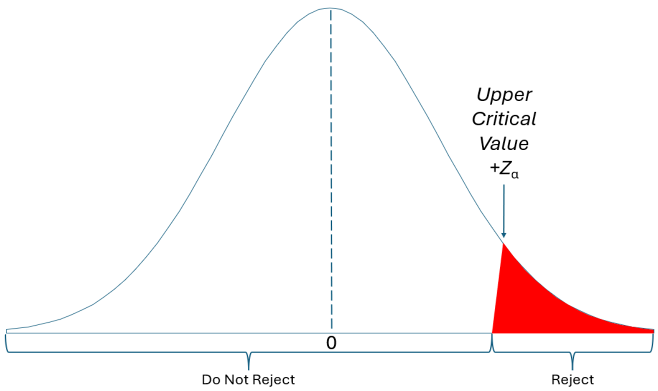
<u>Critical Values Results:</u>

Lower = **-1.96**, Upper = **+1.96**

One Sample Z Upper-Tail Critical Value:

H_0: $\mu \leq \mu_0$

H_1: $\mu > \mu_0$



Calculator Instructions:

From the main menu select:

STAT > **F5**(DIST) > **F1**(NORM) > **F3**(InvN)

Enter the following in Inverse Normal:

Data: **F2** (Var)

Tail: **Right**

Area: **0.05** (level of significance)

σ: **1**

μ: **0**

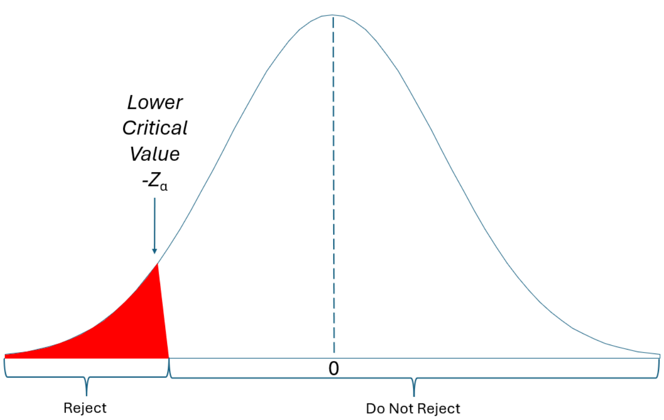
Critical Value Results:

Upper = **+1.64**

One Sample Z Lower-Tail Critical Value:

H_0: $\mu \geq \mu_0$

H_1: $\mu < \mu_0$



Calculator Instructions:

From the main menu select:

STAT > **F5**(DIST) > **F1**(NORM) > **F3**(InvN)

Enter the following in Inverse Normal:

Data: **F2** (Var)

Tail: **Left**

Area: **0.05** (level of significance)

σ: **1**

μ: **0**

Critical Value Results:

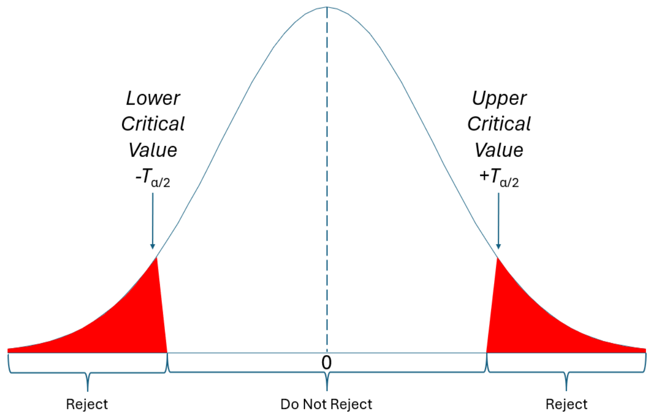
Lower = **-1.64**

When finding critical values for T distributions, the degrees of freedom (df) is equal to n (the sample size) minus 1. The following examples assume a 0.05 level of significance and n = 15.

One Sample T Two-Tail Critical Values:

H_0: $\mu = \mu_0$

H_1: $\mu \neq \mu_0$



Calculator Instructions:

From the main menu select:

STAT > **F5**(DIST) > **F2**(T) > **F3**(InvT)

Enter the following in Inverse T:

Area: **0.05 / 2** (level of significance / 2)

df: **15 - 1**

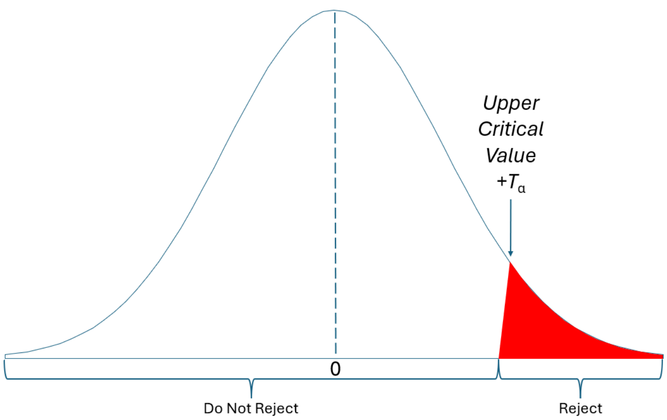
Critical Value Results:

Lower = **-2.14**, Upper = **+2.14**

One Sample T Upper-Tail Critical Value:

H_0: $\mu \leq \mu_0$

H_1: $\mu > \mu_0$



Calculator Instructions:

From the main menu select:

STAT > **F5**(DIST) > **F2**(T) > **F3**(InvT)

Enter the following in Inverse T:

Area: **0.05** (level of significance)

df: **15 - 1**

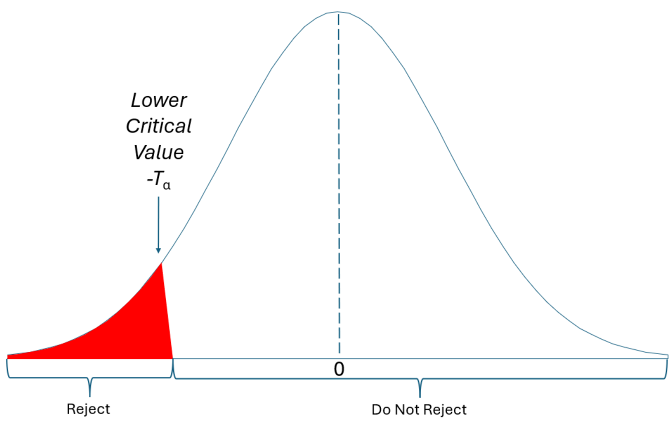
Critical Value Results:

Upper = **+1.76**

One Sample T Lower-Tail Critical Value:

H_0: $\mu \geq \mu_0$

H_1: $\mu < \mu_0$



Calculator Instructions:

From the main menu select:

STAT > **F5**(DIST) > **F2**(T) > **F3**(InvT)

Enter the following in Inverse T:

Area: **1 - 0.05** (1 - level of significance)

df: **15 - 1**

Critical Value Results:

Lower = **-1.76**

Decision Rules:

- **Based on Critical Values:** Compare the test statistic (Z or t) to the critical value(s).
  - For a lower tail test: Reject the null hypothesis if the test statistic is less than the critical value.
  - For an upper tail test: Reject the null hypothesis if the test statistic is greater than the critical value.
  - For a two-tailed test: Reject the null hypothesis if the test statistic is less than the lower critical value or greater than the upper critical value.
- **Based on p-values:** Compare the p-value to the significance level (α).
  - Reject the null hypothesis if the p-value is less than the significance level ($p < \alpha$).
  - Do not reject the null hypothesis if the p-value is greater than or equal to the significance level ($p \geq \alpha$).
  - Remember the rhyme "**If p is low, null must go**".

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Conducting a Z-Test:

Calculator Instructions:
From the main menu select:
STAT > **F3**(TEST) > **F1**(Z) > **F1**(1-S)

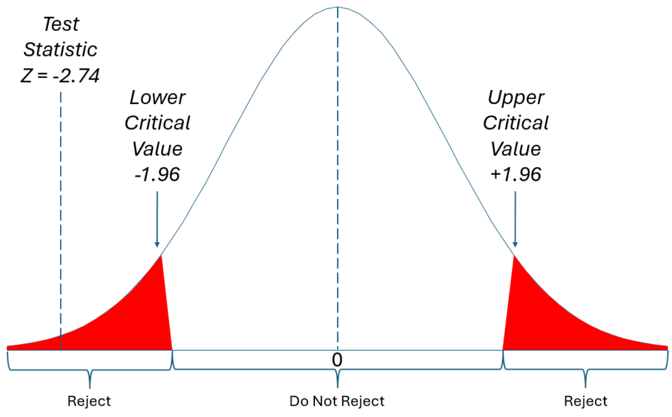
Enter the following:
Data: **F2** (Var)
μ: alternative hypothesis condition (≠, <, >)
μ_0: hypothesized average
σ: population standard deviation
$\bar{X}$: sample average
n: sample size

H_0: μ = 500
H_1: μ ≠ 500

Example 1: A factory claims their product weighs 500 grams on average. A sample of 30 products has an average weight of 495 grams with a **population** standard deviation of 10 grams. Test if the average weight is **different** from 500 grams at a 5% significance level.

Test Instructions:

Enter the following:	Results:
Data: **F2** (Var)	μ ≠ 500
μ: **≠**	Z = -2.74
μ_0: **500**	p = 6.17E-3
σ: **10**	$\bar{X}$: 495
$\bar{X}$: **495**	n: 30
n: **30**	



Critical Value Instructions:
From the main menu select:
STAT > **F5**(DIST) > **F1**(NORM) > **F3**(InvN)

Enter the following in Inverse Normal:
Data: **F2** (Var)
Tail: **CNTR**
Area: **0.95** (1 - level of significance)
σ: **1**
μ: **0**

Critical Values Results:
Lower = **-1.96**, Upper = **+1.96**

Example Breakdown:

The question states the population standard deviation which tells us that it is a Z-test. The question also uses the keyword "different" which tells us that this is a two tailed test.

The test statistic Z = -2.74 is more extreme than the critical value $-Z_{\alpha/2}$ = -1.96 and thus falls in the rejection region on the distribution. The p value 6.17E-3 which is notation for 0.00617 is also much smaller than the level of significance of 0.05 so we reject the null hypothesis.

Conducting a T-Test:

Calculator Instructions:
From the main menu select:
STAT > **F3**(TEST) > **F2**(T) > **F1**(1-S)

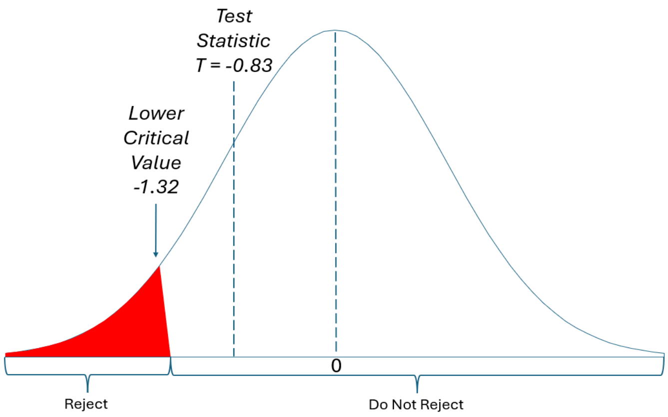
Enter the following:
Data: **F2** (Var)
μ: alternative hypothesis condition (≠, <, >)
μ_0: hypothesized average
S_x: sample standard deviation
$\bar{X}$: sample average
n: sample size

H_0: μ ≥ 600
H_1: μ < 600

Example 2: A factory claims their product weighs 600 grams on average. A sample of 25 products has an average weight of 598 grams with a **sample** standard deviation of 12 grams. Test if the average weight is **less** than 600 grams at a 10% significance level.

Test Instructions:

Enter the following:	Results:
Data: **F2** (Var)	μ < 600
μ: **<**	T = -0.83
μ_0: **600**	p = 0.21
S_x: **12**	S_x: 12
$\bar{X}$: **598**	$\bar{X}$: 598
n: **25**	n: 25



Critical Value Instructions:
From the main menu select:
STAT > **F5**(DIST) > **F2**(T) > **F3**(InvT)

Enter the following in Inverse T:
Area: **0.9** (1 - level of significance)
df: **25 - 1**

Critical Value Results:
Lower = **-1.32**

Example Breakdown:

Since the question does not provide the population standard deviation but instead supplies the sample standard deviation this tells us that this is a T-Test. The question also uses the keyword “less” which tells us that this is a lower tailed test.

The test statistic T = -0.83 is less extreme than the critical value T_α = -1.32 and thus falls in the non-rejection region on the distribution. The p value 0.21 is larger than the level of significance of 0.10 so we do not reject the null hypothesis.

Conducting a Proportion-Test:

Calculator Instructions:

From the main menu select:

STAT > **F3**(TEST) > **F1**(Z) > **F3**(1-P)

Enter the following:

Prop: alternative hypothesis condition (≠, <, >)

P_0: proportion as a decimal

x: sample proportion

n: sample size

H_0: π ≤ 0.60

H_1: π > 0.60

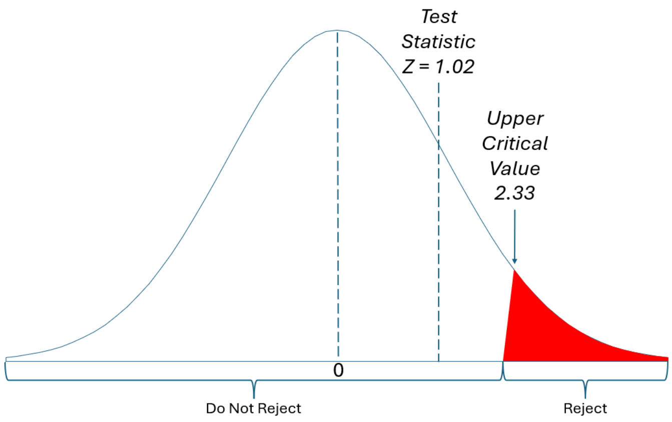
Example 3: A survey claims that 60% of people prefer coffee over tea. In a sample of 100 people, 65% prefer coffee. Test if the percentage of people who prefer coffee is **greater** than 60% at a 1% significance level.

Condition 1: 100 * 0.60 = 60 > 5 ✓

Condition 2: 100 * (1 - 0.60) = 40 > 5 ✓

Test Instructions:

Enter the following:	Results:
Prop: **>**	Prop > 0.6
P_0: **0.6**	Z = 1.02
x: **65** (100 * 65%)	p = 0.15
n: **100**	p̂ = 0.65
	n = 100



Critical Value Instructions:

From the main menu select:

STAT > **F5**(DIST) > **F1**(NORM) > **F3**(InvN)

Enter the following in Inverse Normal:

Data: **F2** (Var)

Tail: **Right**

Area: **0.01** (level of significance)

σ: **1**

μ: **0**

Critical Value Results:

Upper = **2.33**

Example Breakdown:

- The question involves proportions (percentages) of people preferring coffee over tea and has a binary outcome meaning people either prefer coffee or tea (no inbetween) which tells us this is a proportion question.
- The question uses the keyword “greater” which tells us that this is an upper tailed test.
- The test statistic T = 1.03 is less extreme than the critical value Z_α = 2.33 and thus falls in the non-rejection region on the distribution. The p value 0.15 is larger than the level of significance of 0.01 so we do not reject the null hypothesis.

Errors in hypothesis testing:

Type I Error: Reject a true null hypothesis

- The probability of a Type I Error is **α** (alpha)
    - Called the level of significance of the test
    - Set by researcher before the test

Type II Error: Failure to reject a false null hypothesis

- The probability of a Type II Error is **β** (beta)

Possible Hypothesis Test Outcomes:

Decision	**H_0 True**	**H_0 False**
Do Not Reject H_0	No Error (True Positive) Probability = 1 - α	Type II Error (False Negative) Probability = β
Reject H_0	Type I Error (False Positive) Probability = α	No Error (True Negative) Probability = 1 - β

Type I and Type II errors cannot occur simultaneously

- A Type I error can only occur if H_0 is true
- A Type II error can only occur if H_0 is false

Example Breakdowns:

- In example 1 we are at risk of making a Type I Error as the decision was to reject H_0
- In example 2 we are at risk of making a Type II Error as the decision was to not reject H_0
- In example 3 we are at risk of making a Type II Error as the decision was to not reject H_0