Misallocation of Resources, Political Connections and External Flows

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Abstract

This paper shows that the current design of foreign aid and loans may impede growth in developing economies with weak political institutions. First, the paper provides empirical evidence that politically connected Pakistani firms pay lower effective taxes and this tax differential increases with the public external debt to GDP ratio. I then develop a political economy model in which agents connected with the government receive lower taxes and barriers to entry in exchange for political support, causing misallocation in the economy. High external flows give the government more room to lower taxes on connected entrepreneurs, which keeps low productivity, connected firms in the market. I calibrate the model to the economy of Pakistan and show that reducing flows by 30% reduces inequality and generates an output gain of 12%. I also show that a similar outcome could be obtained by adding conditions to existing external flows that require a higher level of fiscal revenues or that reduce barriers to entrepreneurship.

JEL Classification: E60, EO10, P16

Keywords: Corporate taxes, Economic development, External Debt, Fiscal revenues, Foreign aid, Misallocation of resources, Political connections, Political economy.

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1 Introduction

One of the main purposes of foreign aid and external debt is to help developing countries that lack funds to achieve economic growth and reduce inequality. However, the impact of foreign aid and external debt on the economic growth of these economies is ambiguous in literature. Some countries, like South Korea, which relied heavily on foreign aid and external debt from the 1960s to 1980s, experienced significant and unparalleled economic growth (Ugwuegbe et al., 2016). In contrast, other countries, such as Pakistan, have struggled to achieve desired economic growth while their dependency on external debt to meet fiscal needs has increased substantially (Hussain et al., 2017). This raises two important questions in development economics: why do some developing countries experience rapid economic growth while others experience slow growth despite receiving high foreign aid and external debt? Additionally, does international aid and debt hinder economic growth in economies with weak political structures? Answering these questions requires further investigation. Meanwhile, evidence suggests that resources in countries with weak institutions are often allocated based on political connections rather than economic potential, resulting in a misallocation of resources. This paper connects the above facts and shows that external loans and aid can reinforce political patronage systems, where a group of individuals benefit in exchange for their support of the elite, and lead to lower economic growth.

The main objective of this paper is to theoretically and empirically test the mechanism mentioned above, using Pakistan as an example. It does so in the following four ways. First, it uses Pakistani data to empirically document preferential tax treatment towards politically connected Pakistani firms, showing that a higher external debt to GDP ratio exacerbates this preferential treatment. Second, the paper develops a theoretical political economy model to rationalize this connection. Third, the paper calibrates the model to Pakistan and performs counterfactual exercises that reduce foreign flows and shows the growth implications of this.

Finally, the paper makes policy recommendations aimed at making the external aid and debt packages more effective in reducing misallocation for developing countries.

The empirical analysis uses firm-level and election data from Pakistan. Using this data, I find significant evidence of politically connected firms paying 8.37% percentage points lower effective tax rates than non-connected firms. I also find that a 10% increase in the Public external debt to GDP ratio, increases this tax differential and further lowers the effective tax rate paid by the politically connected firms by 9.26% percentage points.

The theoretical analysis proposes a model where the entrepreneurial skills of the individuals evolve overtime. In the model the patronage system consists of preferential treatment received by politically connected firms. This preferential treatment is in terms of paying lower effective tax rates and bypassing the existing barriers to entrepreneurship. This creates misallocation of resources by encouraging low productivity, politically connected firms to produce in the economy. In return for this privilege, the Elite increases its ability to remain in power. Thus, by manipulating tax rates on the connected, government can choose between high tax - no misallocation and low tax - misallocation equilibrium. In this environment, higher foreign aid and external debt flows allow the government to keep the taxes on connected entrepreneurs low which, in turn, can sustain a significant fraction of low-productivity connected firms in operation. In particular, the dynamics of the model show that overtime if the measure of low skilled individuals increase, higher foreign aid and external debt (external flows) worsens the output of the economy. This results in an equilibrium with misallocation and lower output.

I calibrate the model to the economy of Pakistan. In the baseline calibration, Pakistan is in an equilibrium with misallocation. Counterfactual simulations show that reducing external flows by 30% results in an equilibrium without misallocation, a 34% reduction in inequality measured by the Gini coefficient, a 32.4% increase in the welfare of the non-connected entrepreneurs and a 12% increase in the steady state output. I also show that a similar outcome is possible by keeping the debt at the same level but making it contingent on achieving a certain level of direct tax revenues or reducing entry barriers to entrepreneurship. The outcomes suggest the need to restructure existing foreign aid and debt packages in three possible directions: (1) reducing the level of unconditional debt and aid or (2) keeping the same level of debt and aid but adding conditions that require the government to either keep a pre-determined level of tax revenue or (3) to reduce entry barriers to entrepreneurship. The first two options cause the Elite to increase tax rates on the connected entrepreneurs to raise revenues and make it unprofitable for low skilled, connected, firms to stay in the market. The third option reduces the entry costs for non-connected, highly skilled entrepreneurs, making it more profitable for them to enter the market and produce.

The rest of the paper is organized as follows. Section 2 reviews the current literature related to this paper. Section 3 outlines Pakistan's political environment and historical dependence on external debt. Section 4 presents the data, methodology and results of the empirical analysis. Section 5 develops a model of misallocation consistent with the empirical findings. Section 6 discusses the calibration of the model. Section 7 presents the simulation results of the baseline model using the calibrated parameters, Section 8 performs counterfactual exercises, Section 9 discusses policy recommendations based on the results of the model and Section 10 concludes.

2 Literature Review

This study contributes to three areas of the literature. The first one is related to studies investigating the relationship between foreign aid, external debt, growth and institutional quality. Despite a vast amount of empirical literature on these topics, there remains little empirical consensus on the relationship between external flows and growth. The first group of studies uses cross selectional and panel data for multiple countries and find little to no robust evidence of the effectiveness of foreign debt and aid on growth (Rajan & Subramanian, 2007; Reinhart & Rogoff, 2010). Reinhart & Rogoff (2011) in their paper find that high levels of external debt to be a precursor of banking crises and even sovereign debt crises. Other studies find that external flows may lead to a decrease in growth of some of the developing economies and that high level of debt is associated with debt overhang problem and a reduction in investment and GDP per capita (Akram, 2011; Choong et al., 2010; Clements et al., 2003; Guei, 2019). Chatterjee et al. (2012) develop a model to show that foreign aid can be fungible and may relax the government's budget constraint, reducing government spending on pubic goods. Using a panel data of 67 countries they also find that 70% of foreign aid is fungible and that in the presence of corruption, fungibility and rent seeking, aid is ineffective in promoting economic growth. A different set of studies find a positive relationship between aid and growth in the presence of good fiscal and trade policies (Burnside & Dollar, 2000).

Furthermore, there exists strong empirical evidence linking quality of institutions and growth and some evidence of foreign aid having a deteriorating effect on institutional quality (Acemoglu et al., 2005; Young & Sheehan, 2014). Acemoglu (2008) develops a closed economy model to show that in the long run in oligarchic institutions are more inefficient, has a high degree of earnings inequality and low aggregate output than a democratic regime. This is because democratic regimes set lower entrepreneurial entry barriers and higher taxes compared to the oligarchic setup. Policies in this model are determined by majoritarian voting by workers in a democracy and by the existing firm owners who make up the Elite in the oligarchic regime. The paper shows that the oligarchic setup allows low productive firms to remain as entrepreneurs. In the paper policy makers are different in both the regimes. Furthermore, the paths of the two regimes are compared independently, without the possibility of a change in misallocation within the same regime. The model developed in this paper is based on Acemoglu (2008) and it extends it to include political connections and endogenous entrepreneurship decisions by the productive agents based on the optimal policy set by an independent Elite which do not have the ability to produce.

This paper is also related to the literature on misallocation. This category of literature focuses on the link between resource misallocation, total factor productivity (TFP) in developing economies, capital accumulation and growth. In one of the pioneer papers, Restuccia & Rogerson (2008) develop a growth model with establishment level heterogeneity in technology to show that misallocation of resources across heterogeneous firms due to preferential policies can have substantial effects on aggregate TFP. In particular, preferential treatment results in capital misalloction and low aggregate TFP and output per worker. In their influential paper Hsieh & Klenow (2009) use microdata from China and India to quantify the extent of misallocation that exist in these countries compared to the United states. They estimate that reallocating capital and labour to equalize marginal products to the level of the United States would result in TFP gains in the manufacturing sector of 30%-50% for China and 40-60% for India. Huneeus & Kim (2018) in their paper study the effects of lobbying activities on resource misallocation and find that a reduction in lobbying activities increases the aggregate productivity in US by 6%. Furthermore, Fattal-Jaef (2022) using a standard model of firm dynamics show that high barriers to entry for firms, cause distortions in allocative efficiency and that the removal of these barriers may lead to productivity gains of 8%.

This paper also relates to a niche of empirical literature identifying and exploring firms having political connections as a mechanism of obtaining preferential treatment. This preferential treatment consists of credit access, possibly at lower interest rates, paying lower effective taxes and having higher default rates irrespective of their productivity(Ashraf et al., 2020; Khwaja & Mian, 2005; Saeed et al., 2019). Faccio (2006) analyzes data from 47 countries and finds that politically connected firms have higher leverage, higher market shares and low performance compared to their non-connected peers. These results are more prominent for less economically developed countries with higher levels of corruption. Following Faccio (2006), Khwaja & Mian (2005) provide evidence from Pakistan using a firm level dataset to show that politically connected firms receive higher credit access and have higher default rates than non-connected firms. They also show that this preferential treatment and political bias is driven by the lending practices of government banks. Akcigit et al. (2023) in their recent paper develop a growth model to show that firms with political connections have lower innovation and productivity is lower. Using data from Italy for the period of 1993-2014 they empirically test the models predictions and find that politically connected firms are less likely to innovate but may have higher rate of survival and revenues. Overall the empirical results show that the loss from the misallocation and lower growth at the aggregate level outweigh the gains from the political connections in terms of higher aggregate profits due to preferential policies.

The above mentioned papers do not develop or show a possible link between political connections and foreign aid and debt. This paper contributes to this gap in the literature by showing the intersection of the different categories identified above and how it relates to misallocation. Specifically, it shows how the effectiveness of the external aid and debt may depend on the privileges received by the political connected. Additionally, the paper shows that these flows may even lead to an increase in this preferential treatment, thus, maintaining or exacerbating the existing misallocation .

3 Pakistan's External Debt and Political Environment

Dependence on external debt

Over the past 25 years the average GDP growth rate for Pakistan has been lowest at approximately 4% compared to its neighboring countries in the Indian subcontinent and has on average received foreign aid flows of about 2% of its GDP (World Bank, 2020). At the same time, in more than a decade, Pakistan has consistently had one of the lowest tax to GDP ratio's in the Asia Pacific region, ranging from 5 to 12 percent of GDP (Fbr, 2018; Heritage, 2020). Pakistan is bestowed with important strategic endowments and has a high potential for growth. However, Pakistan's growth perspectives have been constantly grappled with poor political governance, high corruption amongst public sector and fiscal mismanagement (Ahmed, 2019). This has lead Pakistan's economic performance to be well below average compared to its comparators in South Asia. Despite worsening of Pakistan's debt sustainability indicators over the years, it has managed to avoid sovereign default and economic crisis by resorting to orthodox stabilization programs provided by the IMF (Ahmed, 2019). In 2020 Pakistan agreed to its 13th bailout by the IMF (International Monetary Fund) in the last 30 years to prevent a severe economic crises and fund its fiscal and trade deficits (Mackenzie, 2019). Currently in 2023 they are once again trying to avert another looming economic crisis and default. They are currently in one of the toughest negotiations with the IMF to obtain another bailout. The conditions of the bailout pertain to increasing their level of current tax revenues and government revenues (Mangi & Dilawar, 2023). Low revenue generation specially from direct taxes due to low number of individuals and companies filing income tax is one of the core structural weakness identified for Pakistan (Cevik, 2018). Despite conditions being placed in the previous bailouts, Pakistan has not been able to bring any meaningful structural change, in particular, to raise tax revenue, and most of the bailouts have been left uncompleted (Mangi, 2023).

Political connections and a system of patronage

Pakistan's fiscal deficit is also exacerbated by inefficient allocation of resources specifically government expenditure, skewed towards promoting and protecting the interests of the political Elite, further stagnating the growth (ISAS, 2020). Pakistani politics is also suffers from severe corruption and politicians often extract rent and distribute it to their patronage network (Jenkins & Kukutschka, 2018). A recent report by UNDP (United nations development program) finds that most of the politicians in the assembly and parliament have considerable business portfolio and are often part of the feudal Elite (UNDP, 2020). The report also finds that tax exemptions made to the feudal Elite from the agricultural sector and the corporate sector constitute 31% and 54% of the total privileges granted to these sectors in the year 2017-2018. Pakistan also suffers from a low rate of tax compliance by the corporate sector, either in the form of under reporting of the formal income or through tax evasion. While 55% of the direct tax revenues for the financial year 2018 were raised from registered businesses in Pakistan, only 56% of the registered businesses with the Securities and Exchange Commission of Pakistan (SECP) filed income tax returns . Out of those companies who filled their income taxes only 7% of them reported income above 7 million PKR (Fbr, 2018; Khan, 2020).

In the next section, using data from Pakistan and Pakistani firms, I estimate the impact of a firm being politically connected on their effective tax rates and the impact of the external debt on the effective tax rates paid by the politically connected firms.

4 Empirical Analysis

This section describes the database, the econometric methodology and presents the results from the empirical analysis. The aim of this econometric analysis is two fold. First, to find a relationship between effective tax rates and the political connectivity of a firm. Second, to establish the relationship between the external debt and the effective tax rates paid by the politically connected firms. I first discuss the dataset related to Pakistan, used to obtain the variables for this analysis (subsection 4.1) and describe the construction of the main variables (subsection 4.2). Then, I describe the econometric methodology used for the estimation (subsection 4.3), followed by the discussion of the main results (subsection 4.4).

4.1 Data sources

The data for the period of 2013-2019 used in this analysis is collected from three main sources. The financial variables and information on the board of directors of the Pakistani firms is taken from the S&P capital IQ database (S & P Capital IQ, 2021). The data for the external debt , GDP and the macro variables used in the model is taken from the World Bank, specifically the external debt related information is taken from the international debt statistics (IDS) country tables for Pakistan (World Bank IDS, 2021). The database of candidates who participated in the 2013 and 2018 general election is constructed using the documents and list available from the Election Commission Pakistan (ECP) (ECP, 2021). It is noteworthy to mention that the candidate name and information is uploaded in a very poorly scanned document on the ECP website, which makes it tedious to obtain the list of the candidates, as the names have to be extracted individually in most cases specifically for the 2013 general elections. The missing data on the financial variables of the firms from the S&P platform is also supplemented from the summary of the annual reports obtained from the State Bank of Pakistan (SPB) and in some cases hand collected from the individual annual reports of each company available on their websites or other sources on the Internet (SPB, 2021).

Sample Selection The process of sample selection start with searching for operational Pakistani publicly listed and private firms with availability of financial reports on the S & P platform as of financial year end 2019. There was information available on 433 public and private operational Pakistani firms as of 2019 out of which 343 were non financial and did not belong to the public sector. Following the common practice in the empirical finance, financial firms are excluded from the sample (Fama & French, 1992)[1]. Out of the 343 firms, 261 listed non financial firms had most of the key financial information available for the period of 2012-2019. Some of the missing key financial variables for these 261 firms were

[1] This is mainly due to differences in the leverage structure of the financial firms, which can impact the sensitivity to interest rate and subsequently their taxable income and valuation

supplemented from the individual company's annual reports available from the State Bank of Pakistan (SPB), their individual websites and some other online sources (SPB, 2021). For the remaining 82 firms these was missing information which cannot be supplemented from other sources. However, for 7 of these firms there was missing information on key variables for some years, which were imputed using the average growth rate of the variables from the prior years. I conduct the empirical analysis using the sample of a balanced panel of 268 firms.

Table 6 in the Appendix A presents the distribution of the sample firms based on their connections.

4.2 Variables Measurement

4.2.1 Firm level: Micro variables

Political Connectedness Following the work of Fama & French (1992) and Khwaja & Mian (2005) I define a politically connected firm as a firm which has one or more members of its board of directors who are politically connected. A member is defined to be politically connected until the next general election if their full name (First, Middle, if applicable and Last) matches with the name of a candidate who took part in the general election.[2] To establish the database on connected board of directors, I use the information available on the key board of directors for the sample firms in the S & P capital IQ database and match it with the database containing the full names of candidates who took part in the general elections of 2013 and 2018 in Pakistan, constructed from the data available with the ECP (ECP, 2021). Some of the firms in the original sample had more than one politically connected board member who took part in a particular year's general election. According to the definition there is no difference between firms having one or multiple board members who are politically connected.

[2]Here, I assume that if any board member of a previously connected firm is not a candidate taking part in the next general election the firm loses its connection, as it is possible that the member is not strongly affiliated with politics or a political party anymore and do not receive any preferential treatment.

In the following analysis a firm's status of political connectedness is represented by a time-variant dummy variable POLCON, which takes value one if the firm has at least one politically connected person in its board of director and zero otherwise.

Measure of tax rates Following previous studies in the empirical finance and accounting literature (Adhikari et al., 2006; Gupta & Newberry, 1997), a firm's tax rate is measured by calculating the effective income tax rate (ETR). A firm's ETR is defined as tax expenses excluding any portion of deferred tax expenses which is not yet paid as a ratio of the profit or earnings before interest and tax (EBIT) (Faccio, 2006). An alternative would be to use operating cash flow as income instead of EBIT (Zimmerman, 1983) [3]. More than 50% of firms in my sample had missing information on operating cash flow for certain years, which can not be supplemented from any other sources. Thus, I adopt the more common practice of using EBIT as the taxable income. The ETR in our sample is calculated as:

$$ETR = \frac{Tax\ expense - Deferred\ taxes}{EBIT}$$

There are some measurement issues which are related to the calculation of ETR as follows; 1) there are firms with negative taxes (tax refunds) 2) there are firms that have positive taxes and negative EBIT and 3) there are firms that have unreasonably small denominator (EBIT) resulting in ETR to be greater than 1 or tax rate to be higher than 100%. Following previous studies (Adhikari et al., 2006; Gupta & Newberry, 1997), I retain these firms in the sample and use the following data cleaning and recoding scheme: 1) set ETR = 0 for firms with tax refunds 2) set ETR =1 for firms with positive taxes and negative income and 3) constrain the ETR ratio of the sample to be between 0 and 1 so that the maximum tax rate

[3] These studies do not show significant difference in their regression analysis obtained by using the operating cash flow compared to EBIT.

is set at 1 for firms with ETR above 1.

Control Variables I control for the firm size using the natural logarithm of total assets (SIZE), for tangible assets using the ratio of fixed assets diveded by the total assets (COLLATERAL) and for firm's profitability or return on assets (ROA) by dividing the firm's EBIT by the total assets. These control variables have been identified as being correlated with firm performance and other financial decisions by empirical studies in finance like Adhikari et al. (2006), Faccio (2006) and Saeed et al. (2019).

4.2.2 Macro Variables

Public External Debt to GDP ratio The total external debt stock of Pakistan includes net loans owed to the IMF, long term public and publicly guaranteed external debt stock , long term private external debt stock, and short term external debt stock for a particular year end (World Bank IDS, 2021). I use the public and publicly guaranteed external debt stock (which includes long term public and publicly guaranteed external debt stock and the short term external debt) and the nominal GDP of Pakistan in dollars to construct the (EDPGDP) ratio for the period of 2013-2019[4].

Based on the premise of this study and observed facts, for politically connected firms, change in the preferential treatment in the form of lower taxes is more likely to be associated with the external debt obtained by the government compared to the external debt received by the private sector. Thus, I use (EDPGDP) for the main analysis. I also use the Total External debt to GDP ratio (EDGDP) for robustness exercises, reported in the Appendix A.

Control Variables I include some macro level control variables which might be correlated with external debt to GDP ratio as well as the overall performance of the firms in the

[4]According to the world bank description of the short term external debt there is currently no accurate procedure to differentiate between private and public short term external debt.

economy. I control for the government spending by using the government spending to GDP ratio (GOVGDP) (World Bank IDS, 2021), lending interest rate measured using the annual average SBP lending interest rate (Lending%) and for the foreign exchange effect using the average annual foreign exchange rate of Pakistani Rupee in terms of US dollars (FX) (SPB, 2021).

Summary statistics for the final variables are provided in Table 7 in the Appendix A.

4.3 Model Estimation

Based on the above premises, I construct the following two Hypotheses for the case of Pakistan:

Hypothesis 1: Ceteris Paribus, politically connected firms pay lower effective tax rates than the non-connected firms.

Hypothesis 2: Ceteris Paribus, the difference between the lower effective tax rates paid by politically connected firms than the non-connected firms, is higher when the public external debt to GDP ratio is high.

I use individual fixed effects estimation technique for our model estimation. Fixed effects estimation is a popular method used for controlling endogenity caused by unobservable, firm specific, variables which might jointly determine financial variables and political connectivity of a firm (Adhikari et al., 2006; Saeed et al., 2019; Wintoki, 2007). The fixed effect model specification to test **Hypothesis 1** is as follows:

$$ETR_{it} = \alpha_0 + \beta_1 \cdot POLCON_{it} + \lambda \cdot X_{it} + \psi_i + \pi_t + \epsilon_{it} \tag{1}$$

where ETR is the effective tax rate of a firm, $POLCON$ is a dummy variable indicating political connectedness, it takes a value 1 if a firm is politically connected in a year, X_{it} are micro firm level control variables consisting of $(SIZE)$, $(COLLATERAL)$ and (ROA), ψ_i is individual firm fixed effects, π_t is time fixed effects and ϵ is the error term.

The fixed effects model specification to test **Hypothesis 2** are as follows:

$$\begin{aligned} ETR_{it} = \alpha_0 + \beta_1 \cdot POLCON_{it} + \beta_2 \cdot EDPGDP_t + \beta_3 \cdot POLCON_{it} * EDPGDP_t \\ + \lambda_1 \cdot X_{it} + \lambda_2 \cdot X_t + \psi_i + \epsilon_{it} \end{aligned} \tag{2}$$

where $EDPGDP$ is the public and publicly guaranteed debt stock as a ratio of the GDP and X_t are macro level control variables including $(GOVGDP)$, $(Lending\%)$ and (FX).

Our main coefficient of interest is β_1 for specification (1). The predicted sign for β_1 is negative which corresponds to with politically connected firms paying lower effective tax rates than their non-connected peers. The main coefficient of interest for specification (2) is β_3 which captures the interaction between a firm being politically connected and EDPGDP on the effective tax paid by the connected firms. The predicted sign for β_3 is negative, if the politically connected firms pay lower effective taxes when EDPGDP increases.

4.4 The Effect of Political Connectedness and Public External Debt to GDP Ratio on Effective Tax Rates

Table 3 shows the regression results for specifications (1) and (2) using the sample of 268 firms. Column (1) of Table 1 shows that the coefficient of POLCON is positive and significant. More precisely the results show that politically connected firms pay 8.37 percentage points lower effective taxes than their non-connected peers, which supports **Hypothesis 1** and is in line with prior studies such as Adhikari et al. (2006) and Saeed et al. (2019). The control variables COLLATERAL and ROA are also significant, showing that as predicted these variables negatively affect the effective tax rates of a firm and is consistent with previous empirical studies (Adhikari et al., 2006; Faccio, 2006; Saeed et al., 2019).

Column (2) of Table 1 shows that the coefficient of the interaction term POLCON*EDGDP is significantly negative . This supports **Hypothesis 2** and shows that a 10 percentage points increase in the public and publicly guaranteed external debt to GDP ratio for Pakistan lowers the effective tax rates paid by the politically connected firms by 9.26 percentage points. This supports our premise that the loans provided to the government of Pakistan result in an increase in the preferential treatment received in terms of lower taxes for the politically connected firms and might not be efficiently allocated to more productive use. Notice that column (2) shows that the coefficient of POLCON is significant but positive, contrary to our results for specification (1). This may not be completely counterintuitive as when an interaction term is added, the coefficient of the interaction term alone is not sufficient to measure the impact on the independent variable. In this case the coefficient of POLCON shows the effect of POLCON on the effective tax rates when the public and publicly guaranteed external debt to GDP ratio is 0. However, as EDPGDP is a continuous variable and is unlikely to be 0 for any of the years, interpreting the coefficient of POLCON by itself in these specifications may not be informative for our analysis.

Table 8 in Appendix A shows the results for specification (1) and (2) using the sample of 261 firms excluding the firms with the extrapolated financial variables as well as specification (2) using the ratio EDGDP instead of EDPGDP. It shows that results are not sensitive to the exclusion of the additional 7 firms. It also reinforces that the findings are consistent with the fact that the coefficient β_3 is higher when there is an increase in the ratio of EDPGDP compared to EDGDP which includes the private long term external debt.

To sum up, the empirical results presented in this section provides evidence of the existence of a significantly high measure of politically connected listed firms in Pakistan. It also confirms that one of the ways these firms receive preferential treatment due to their connectivity with the political parties is through paying lower effective tax rates than their non-connected peers and that this preferential treatment is higher when the external debt to GDP ratio increases.

Table 1: Regression table

	(1)	(2)
	ETR	ETR
POLCON	-0.0837**	0.168**
	(0.0339)	(0.0857)
EDPGDP		0.0858
		(0.747)
POLCON*EDPGDP		-0.926***
		(0.290)
SIZE	-0.00458	-0.0273
	(0.0390)	(0.0406)
COLLATERAL	-0.150**	-0.159**
	(0.0656)	(0.0639)
ROA	-0.293***	-0.270***
	(0.0651)	(0.0657)
GOVGDP		-4.885
		(6.600)
LENDING(%)		-1.200
		(0.807)
FX		0.00199
		(0.00205)
Firm fixed effects	Yes	Yes
Time fixed effects	Yes	No
No of Observations	1876	1876

Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Variable definitions: ETR = (Tax expenses- Deferred tax expenses)/(Earnings before interest rate and tax); POLCON=1 if the firm has a board of director who is politically connected; 0 otherwise; EDPGDP = (public and publicly guaranteed external long term debt stock+ short term external debt stock in US dollars)/ (Nominal GDP in US dollars); SIZE= Log of Total Assets; COLLATERAL= (Total Assets-Total current Assets)/(Total Assets); ROA= (Earnings before interest and tax)/(Total Assets); GOVGDP= (Total government expenditure in US dollars)/(Nominal GDP in US dollars); Lending(%)= Annual average SBP lending interest rate; FX= Average annual foreign exchange rate of Pakistani Rupee in terms of US dollars.

In what follows I develop a theoretical model that is consistent with the two previous facts and I use it to illustrate the main mechanism leading to misallocation under certain political environment. I then calibrate the model to Pakistan to obtain more quantitative results regarding the extent of the misallocation. I then perform counterfactual experiments to determine the effects of changing the levels of external debt and foreign aid on the misallocation, measured by the total output of the economy.

5 The Model

In this section I develop a dynamic political economy model with external flows that provides, the mechanism that is consistent with the empirical results in the previous section. Subsection 5.1 describes the environment of the model and subsection 5.2 characterizes the economic equilibrium and provides the solution of the economic and political equilibrium.

5.1 Environment

I consider a political economy model in the spirit of Acemoglu (2008). The model consists of an infinite horizon small open economy populated by one Elite who is in power and a continuum of risk neutral agents. In this economy agents are heterogeneous and they differ in three dimensions: political connectedness with the Elite, entrepreneurial ability and existing entrepreneurship status. Connected agents receive certain perks and patronage in return for their support of the Elite's ability to stay in power. These benefits constitute of (1) receiving a fixed transfer payment from the Elite, (2) being exempt from costly entry barriers into entrepreneurship and (3) receiving more favorable corporate taxation policies. Higher entry barriers to entrepreneurship for non-connected agents can be interpreted as bribes taken by the public officials, the cost of bureaucratic procedures such as delays in getting licenses, permits and contracts, harassment by public officials, restricted access to credit and other such forms of rent extraction (Desai et al., 2011). The barriers to entrepreneurship are constant and these entry barriers do not directly benefit the Elite. However, the politically connected agents have the privilege to bypass them when entering as entrepreneurs.

Every period the agents in the model make occupational decisions. In particular, they choose whether to become a workers or an entrepreneurs. In the model only the entrepreneurs have the ability to produce and the Elite makes all the policy decisions. I assume that only a small fraction of the agents have connections, so that in equilibrium some non-connected agents always become entrepreneurs. Entrepreneurs face a corporate tax imposed by the Elite, the magnitudes of which differs between connected and non-connected entrepreneurs.

The Elite chooses the period taxes for the connected and non-connected entrepreneurs in order to maximize their own lifetime utility. Each period it receives income in the form of tax revenues from the entrepreneurs and external flows, specifically external debt and foreign aid. The Elite's ability to remain in power is a function of the level of support from the connected agents, which is directly related to the tax rate set on connected entrepreneurs.

External flows, which are exogenously given in the model, have an important impact on the optimal tax rates set by the Elite. High external flows provide more revenues and room for the Elite to set lower taxes for the connected entrepreneurs, simultaneously increasing their probability of staying in power. Lower taxes allow low skilled connected entrepreneurs to enter or remain in the market, crowding out the entry opportunity of the more productive high skilled non-connected entrepreneurs. This results in resource misallocation, as resources are being inefficiently utilized in production by low productivity entrepreneurs, affecting the distribution of wealth and potentially the total output of the economy.

5.1.1 Agents

Distribution of agents There is a continuum of agents in the economy. An agent's type is determined by three characteristics: connectivity, entrepreneurial ability and existing entrepreneurship status. The connectivity of each agent is denoted by $j \in \{c, n\}$ as being with connections (c) or no connections (n). Entrepreneurial ability is determined by the level of skills, high (H) or low (L) and the ability is denoted by z. A^z is the productivity of an agent with ability $z \in \{H, L\}$ with $A^L < A^H$. Existing entrepreneurship status is given by $s \in \{e, r\}$, existing firm owner (e) no firm owner (r). The entrepreneurial ability and entrepreneurship status may change overtime, whereas connectivity is a permanent trait. Formally, I denote an agent's type at the start of the period t by m with $m = jzs$.

I assume that a fraction $\phi^c \in (0, 1)$ of agents are connected. Initially a fraction $M_0^H \in (0, 1]$ are high skilled agents. At the start of each period a fraction θ of entrepreneurs face an exogenous shock and exit from entrepreneurship, irrespective of their entrepreneurial ability or connectivity, and become a worker. The entrepreneurial ability z evolves over time following the process below.

$$z_{t+1} = \begin{cases} H & \text{with probability} \quad \sigma^H \text{ if } z_t = H \\ H & \text{with probability} \quad \sigma^L \text{ if } z_t = L \\ L & \text{with probability } 1 - \sigma^H \text{ if } z_t = H \\ L & \text{with probability } 1 - \sigma^L \text{ if } z_t = L \end{cases} \tag{3}$$

where $\sigma^H, \sigma^L \in (0, 1)$. Here σ^H is the probability that an agent is high skilled in period $t+1$ conditional on being high skilled in period t and σ^L is the probability that an agent is high skilled in period $t+1$ conditional on being low skilled in period t. Following Acemoglu (2008) I assume that, $\sigma^H > \sigma^L$. This states that the comparative advantage of entrepreneurs may change over time. As existing high skilled entrepreneurs can become low skilled or vice versa, either due to the inconsistency of skills of an individual overtime or due to the evolution of the importance of the industry or market that the skill set of the entrepreneur belongs.

Ownership status depends on the previous period's occupational choice. The transition for the firm ownership of an agent s_t evolves according to a simple rule as follows: if an agent jz decides to be an entrepreneur and operate a firm at time t, provided that he did not receive the exit shock and become a worker at the start of period $t+1$, then he will be an incumbent entrepreneur with $s_{t+1} = e$ in the next period, otherwise he will be a non existing owner with $s_{t+1} = r$.

For simplicity I also assume that the initial fraction of existing entrepreneurs is zero. This fraction is independent of skill and connectivity.

With this notation, the initial distribution of agents among different types, N_0^m is then as follows:

(i) $N_0^{cHr} = \phi^c M_0^H$

(ii) $N_0^{cLr} = \phi^c(1 - M_0^H)$

(iii) $N_0^{nHr} = (1 - \phi^c)M_0^H$

(iv) $N_0^{nLr} = (1 - \phi^c)(1 - M_0^H)$

(v) $N_0^{cHe} = N_0^{cLe} = N_0^{nHe} = N_0^{nLe} = 0$

I make the following simplifying assumptions affecting the distribution of agents:

Assumption 1. *It is the case that:*

(a) The size of the firm is fixed so that all firms hire the same measure of workers $\bar{L}$.

(b) The measure of connected agents satisfies $\phi^c < \frac{1}{\bar{L}}$.

(c) If all high skilled agents become entrepreneurs, they generate more than sufficient demand to employ the entire labor supply at any given period, so that $M_0^H \bar{L} > 1$ and $\bar{\sigma}\bar{L} > 1$, where $\bar{\sigma} = \frac{\sigma^L}{1-\sigma^L+\sigma^H}$.

(d) *Starting form any* M_0^H *there are always some non-connected high skilled new entrepreneurs. So that the measure of firms dying every period satisfies* $\phi^c + \sigma^H(1-\theta)(\frac{1}{L} - \frac{1}{L}\phi^c) < \frac{1}{L}$ *and* $(1-\sigma^H) > \sigma^L$.

I make this assumption to focus on particular situations that are more relevant in showing the misallocation and are closer to the world economies.

Assumption 1(a) implies that in equilibrium every period the measure of entrepreneurs in the economy is $\frac{1}{L}$ and the assumption 1(b) implies that the total measure of connected agents is less than the equilibrium total measure of firms and, therefore, there are always some non-connected entrepreneurs in the economy..

In what follows I show that the evolution of the measure of high skilled agents M_t^H over time, is determined by exogenous parameters in the model. Where the transition rule for M_t can be simplified as $M_{t+1}^H = \sigma^H M_t^H + \sigma^L(1-M_t^H)$ and converges to a stationary point.

Lemma 5.1. *Let* M_t^H *be the measure of high skilled agents in period* t*. The sequence has a stationary point where* $M^H = \bar{\sigma}$.

Proof. Given the evolution of skills described in expression 3, we can write the measure of high skilled agents M_{t+1}^H at period $t+1$ recursively as:

$$M_{t+1}^H = M_t^H - (1-\sigma^H)M_t^H + \sigma^L(1-M_t^H)$$

A stationary point satisfies

$$M_{t+1}^H = M_t^H = M^H \quad \forall t$$

Then

$$M^H = \sigma^H M^H + \sigma^L(1-M^H)$$

$$M^H = \frac{\sigma^L}{(1-\sigma^H+\sigma^L)} = \bar{\sigma}$$

$\square$

The next lemmas show that for any initial measure of high skill agents, the sequence converges monotonically $\bar{\sigma}$.

Lemma 5.2. *Given M_0^H an initial measure of high skilled agents at time 0. Then $\forall\ M_t^H \in (0,1]$, $M_0^H \underset{t\to\infty}{\to} \bar{\sigma}$.*

Proof. Given M_0^H let us compute:

$$M_1^H = \sigma^H M_0^H + \sigma^L(1 - M_0^H)$$
$$M_2^H = M_0^H(\sigma^H - \sigma^L)^2 + \sigma^L(\sigma^H - \sigma^L) + \sigma^L$$
$$M_3^H = M_0^H(\sigma^H - \sigma^L)^3 + \sigma^L(\sigma^H - \sigma^L)^2 + \sigma^L(\sigma^H - \sigma^L) + \sigma^L$$

In general

$$M_t^H = M_0^H(\sigma^H - \sigma^L)^t + \sigma^L \sum_{s=0}^{t-1}(\sigma^H - \sigma^L)^s$$

Taking the limits as $t \to \infty$, the first term converges to zero given that $(\sigma^L, \sigma^H) \in (0,1)$ and $\sigma^H > \sigma^L$ and the second term converges to $\frac{\sigma^L}{1-\sigma^H+\sigma^L}$, given that $(\sigma^H - \sigma^L) \in (0,1)$. Therefore M_t converges to $\frac{\sigma^L}{1-\sigma^H+\sigma^L} = \bar{\sigma}$ $\square$

Lemma 5.3. *The sequence M_t^H is monotone. In particular, (i) if $M_0^H < \bar{\sigma}$ then M_t^H is a strictly increasing sequence, (ii) if $M_0^H > \bar{\sigma}$ then M_t^H is a strictly decreasing sequence and (iii) if $M_0^H = \bar{\sigma}$ then M_t^H is a constant sequence.*

Proof. (i) To prove (i) I will first show that $M_t^H < M_{t+1}^H$, when $M_0^H < \bar{\sigma}$ and then show that $M_t^H \leq M_{t+1} \leq \bar{\sigma}$.

$M_1^H = \sigma^L + (\sigma^H - \sigma^L)M_0^H$. We know that $M_0^H < \bar{\sigma}$. Thus, substituting in for the value of $\bar{\sigma}$, $M_0^H < \frac{\sigma^L}{1-\sigma^H+\sigma^L}$ and then rearranging it $M_0^H < \sigma^L + (\sigma^H - \sigma^L)M_0^H$, where

$\sigma^L + (\sigma^H - \sigma^L)M_0^H = M_1^H$. Therefore $M_0^H < M_1^H$ and $M_t^H < M_{t+1}^H$.

Now I show that $M_{t+1} < \bar{\sigma}$. Proof by contradiction. Lets suppose $M_1^H \geq \bar{\sigma}$. Then:

$$\sigma^L + (\sigma^H - \sigma^L)M_0^H \geq \frac{\sigma^L}{(1 - \sigma^H + \sigma^L)}$$
$$= M_0^H \geq \frac{\sigma^L}{(1 - \sigma^H + \sigma^L)}$$
$$= M_0^H \geq \bar{\sigma}$$

which is a contradiction $M_0^H < \bar{\sigma}$. So that by induction we obtain $M_t^H \leq M_{t+1}^H \leq \bar{\sigma}$.

(ii) To prove (ii) I will first show that $M_t^H > M_{t+1}^H$, when $M_0^H > \bar{\sigma}$ and then show that $\bar{\sigma} \leq M_{t+1}^H \leq M_t$.

$M_1^H = \sigma^L + (\sigma^H - \sigma^L)M_0^H$. We know that $M_0^H > \bar{\sigma}$.Thus, substituting in for the value of $\bar{\sigma}$, $M_0^H > \frac{\sigma^L}{1-\sigma^H+\sigma^L}$ and then rearranging it $M_0^H > \sigma^L + (\sigma^H - \sigma^L)M_0^H$, where $\sigma^L + (\sigma^H - \sigma^L)M_0^H = M_1^H$. Therefore $M_0^H > M_1^H$ and $M_t^H > M_{t+1}^H$.

Now I show that $M_{t+1} > \bar{\sigma}$. Proof by contradiction. Lets suppose $M_1^H \leq \bar{\sigma}$. Then:

$$\sigma^L + (\sigma^H - \sigma^L)M_0^H \leq \frac{\sigma^L}{(1 - \sigma^H + \sigma^L)}$$
$$= M_0^H \leq \frac{\sigma^L}{(1 - \sigma^H + \sigma^L)}$$
$$= M_0^H \leq \bar{\sigma}$$

which is a contradiction $M_0^H > \bar{\sigma}$. So that by induction we obtain $\bar{\sigma} \leq M_{t+1}^H \leq M_t^H$.

(iii) Show $M_0^H = M_1^H$ and $M_t^H = M_{t+1}^H = \bar{\sigma}$

$M_1^H = \sigma^L + (\sigma^H - \sigma^L)M_0^H$. We know that $M_0^H = \bar{\sigma}$. Substituting for $M_0^H = \bar{\sigma}$, $M_1^H = \sigma^L + (\sigma^H - \sigma^L)(\frac{\sigma^L}{(1-\sigma^H+\sigma^L)}$. Rearranging we get $M_1^H = \frac{\sigma^L}{(1-\sigma^H+\sigma^L)} = M_0^H = \bar{\sigma} = M^H$.

$\square$

Agents' Decisions Let (B^c, B^n) be the exogenous entry barriers for the connected and non-connected entrepreneurs. Let $p_t = (\tau_t^c, \tau_t^n)$ be the vector of policies at time t and let $p^t = \{p_n\}_{n=t}^{\infty}$. Agents at time t make decisions taking the sequence of tax policies p^t, sequence of wages w^t and (B^c, B^n) as given. To simplify on notation and economize on space, I omit the dependence of choice variables on policies in what follows.

In this economy the expected lifetime utility of an agent type m is then given by the following preferences:

$$U_0^m = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t c_t^m, \tag{4}$$

where $c_t^m \in \mathbb{R}$ denotes consumption of an agent of type m and $\beta < 1$ is the discount factor.

Each period an agent makes the occupational decision x_t on whether to work as an entrepreneur ($x_t = V$) or to be employed as a production worker ($x_t = W$). Entrepreneurs make an investment decision $k_t \geq 0$ and a labor input decision $\ell_t \geq 0$. Individuals who decide to become new entrepreneurs face costly entry barriers B^j. I normalize this cost to $B^c = 0$ for the connected entrepreneurs and assume $B^n > 0$ for the non-connected entrepreneurs. With this normalization, there is no difference between the problem and decisions of existing and new connected entrepreneurs. Hence forth, to save on notation I denote a particular type of connected entrepreneur as $m = cz$ and omit the entrepreneurship status indicator.

Technology Each period entrepreneurs produce a single non-storable final good denoted by y_t. As in Acemoglu (2008) the entrepreneur itself works as one of the production workers in the firm, which implies that the opportunity cost of becoming an entrepreneur is 0. An entrepreneur with skill level z can produce using the following production technology:

$$y_t = \frac{1}{1-\alpha}(A_t^z)^{\alpha}(k_t)^{1-\alpha}(\ell_t)^{\alpha} \tag{5}$$

where $\alpha \in (0,1)$ represents the income share of labor.

For simplicity also as in Acemoglu (2008), I allow for negative consumption if an entrepreneur wants to invest more than its output in a given period. This implies that the cost or the price of capital relative to output is equal to 1 in equilibrium. As mentioned in assumption 1 (b) the number of workers hired by each firm is fixed at $\bar{L}$, thus, $\ell_t = \bar{L}\ \forall\ t$.

Finally, I assume that each entrepreneur must operate their own firm and delegation to high skilled managers is prohibitively costly. So the entrepreneur's skill z matters for output. Without the latter assumption entry barriers would create no distortions.

5.1.2 Elite

The Elite is a separate entity from the agents. The Elite is in power and sets the sequence of taxes p^t, with $\tau^c, \tau^n \in (0, \alpha)$. [5] I justify this upper bound by assuming that entrepreneurs can hide their will result in no tax revenues for the Elite as in Acemoglu (2008). The Elite announces a sequence of tax levels at period 0, before agents make any decisions and it sets the actual tax rate every period t after the agents make their occupational, labor input and investment decisions based on the anticipated policies. The Elite does not have the ability to produce and they receive income in the form of tax revenues and fixed external inflows (foreign aid F and external debt D). I assume that every period the Elite is able to issue a fixed amount of foreign bonds to raise external debt, which is paid with interest by the rest of the agents in the following period. Thus, in any period t the economy holds a current account deficit as is the case for most of the developing small open economies.

The Elite pays patronage to maintain the connections and to remain in power. The Elite can

[5] Where, I assume $\bar{\tau}$ to be the upper bound on taxes, with $\bar{\tau} = \alpha$. This is just for simplification. Even without this assumption given the lifetime utility and the budget constraint of the elite and other assumptions on the parameters of the model, it can be shown that the optimal $\tau_t^n = \alpha$.

be overthrown with some probability $\delta(\tau_t^c, \tau_t^n)$, which depends on the tax rate τ_t^c and τ_t^n , for the connected and non-connected entrepreneurs. Besides the preferential tax rates, every period the Elite also pays a fixed lump-sum transfer $P^c > 0$ for each connected agent and sets entry barriers fixed $B^n > 0$ for non-connected entrepreneurs. For simplicity I consider P^c and B^n as exogenously given and constant overtime.

The expected utility of the Elite at time 0 is given by:

$$\tilde{U_0} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (1 - \delta(\tau_t^c, \tau_t^n)) \tilde{c}_t \tag{6}$$

where $\tilde{c}$ denotes the consumption of the Elite.

I assume that if the taxes on the connected entrepreneurs are greater than the non-connected entrepreneurs $\tau_t^c > \tau_t^n$ or if $\tau_t^n < \alpha$ then there will be no support from the connected towards the Elite to remain in power. Therefore, in equilibrium the Elite will always choose $\tau_t^n = \alpha$ and so I will abuse the notation and write $\delta(\tau_t^c, \tau_t^n)$ as $\delta(\tau_t^c)$ and assume $\tau_t^n = \alpha$ in what follows. Where, $\delta(\tau_t^c)$ is an increasing function of τ_t^c and $\delta'(\tau_t^c) > 0$.

Market Clearing Labor market clearing requires the total demand for labour to be equal to the supply. Given that entrepreneurs also work as production workers in their own firm, the total supply of labour is equal to one at any point in time.

Let N_{Vt}^m be the measure of entrepreneurs of type m in equilibrium in period t.

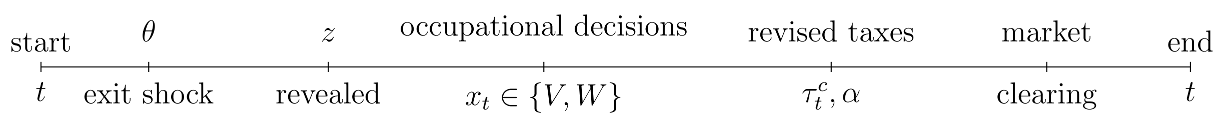
The market clearing condition is given by:

$$\bar{L} \cdot \sum_{\substack{z \in \{H,L\} \\ s \in \{e,r\}}} (N_{Vt}^{nzs} + N_{Vt}^{czs}) = 1. \tag{7}$$

Which implies that in period t in equilibrium, the measure of entrepreneurs is $\frac{1}{L}$.

Timing of Events

The timing of the events in this economy is as follows: At the start of the economy at period 0 the Elite announces a sequence of policies p^t. Every period t the existing entrepreneurs face the exogenous exit shock and a fraction θ of them lose their existing entrepreneurship status. Each agent's entrepreneurial skill z is realized. All agents make their individual occupational choices x_t, entrepreneurs make investment decisions k_t contingent on the announced sequence of taxes. Elite decides whether to revise the announced period tax rates τ_t^c and $\tau_t^n = \alpha$ on entrepreneurs. The labor and goods market clears and wage w_t is determined.



5.2 Equilibrium

I divide the definition of the equilibrium in two parts: one involving the agents' choices, given the announced policy sequence (economic equilibrium) and the policy choice by the Elite, taking into account how its policy choices affect agents' choices (policy equilibrium).

5.2.1 Economic Equilibrium

Given the timing of the events agents know the sequence of announced policies p^t before making occupational decisions . The economic equilibrium solves the agents problem based on the expected policies p^t. In equilibrium given consistency in beliefs the annouced taxes are equal to the actual optimal taxes set by the Elite.

Profit Maximization Let us define $b^j = B^j/\bar{L}$ as the per worker entry cost, where $b^c = 0$ for the connected agents.

Given wages w_t and expected policies τ_t^c, τ_t^n and given the fact that $\ell_t = \bar{L}$ the profit return gross of the cost of entry barrier of an entrepreneur of type jz is then given by:

$$\pi_t^{jz}(k_t^{jz}, \tau_t^j, w_t) = \frac{1-\tau_t^j}{1-\alpha}(A^z)^\alpha (k_t^{jz})^{1-\alpha}(\bar{L})^\alpha - w_t\bar{L} - k_t^{jz} \tag{8}$$

The net gain to an entrepreneur who has to pay entry cost $b^j\bar{L}$ is then given by:

$$\pi_t^{jz}(k_t^{jz}, \tau_t^j, w_t) - b^j\bar{L}$$

Note that since we have normalized the fixed entry cost for the agents with connections to zero, their net gain is always equal to their gross profits.

The profit maximizing optimal investment by an entrepreneur of type jz is then given by:

$$({k_t}^{jz})^* = (1-\tau_t^j)^{1/\alpha} A^z \bar{L} \tag{9}$$

Notice that the optimal investment is higher for $z = H$ and decreasing in the tax rate.

In equilibrium the optimal profit gross of the cost of the entry barriers is then given by replacing (9) into (8):

$$\pi_t^{jz}(\tau_t^j, w_t) = \frac{\alpha}{1-\alpha}(1-\tau_t^j)^{1/\alpha} A^z \bar{L} - w_t\bar{L} \tag{10}$$

Occupational Decisions Agents choose the occupation that maximizes their expected lifetime utility at time t. Given the sequence of policies and wages $q^t = (p^t, w^t)$, I define V_t^m

as the value function of becoming an entrepreneur at time t gross of entry costs and W_t^m as the value function of being a worker at time t. Then I can write the expected lifetime utility of the agent at time t as:

$$U_t^{jzs} = \max\{V_t^{jz} - \mathbb{I}_{\{s=r\}} b^j \bar{L}, W_t^{jz}\} \tag{11}$$

where the indicator variable $\mathbb{I} = 1$, if the agent is a new entrepreneur and $\mathbb{I} = 0$, if the agent is an existing entrepreneur as only the new entrepreneurs pay the entry barrier.
The value function of being an entrepreneur at time t for type jz gross of entry costs is:

$$V_t^{jz} = w_t + P^j + \pi_t^{jz}(\tau_t^j, w_t) - D_{t-1}(1 + r^*) + \beta((1-\theta)(CV_{t+1}^{jz}) + \theta(CW_{t+1}^{jz})) \tag{12}$$

where CV_{t+1}^{jz} is the continuation value of an entrepreneur of type jz at period t entering period $t+1$ as an entrepreneur: $(s_{t+1} = e)$:

$$CV_{t+1}^{jz} = \sigma^z \max\{W_{t+1}^{jH}; V_{t+1}^{jH}\} + (1-\sigma^z)\max\{W_{t+1}^{jL}; V_{t+1}^{jL}\} \tag{13}$$

and CW_{t+1}^{jz} is the continuation value of a worker of type jz at period t entering period $t+1$ as a worker: $(s_{t+1} = r)$:

$$CW_{t+1}^{jz} = \sigma^z \max\{W_{t+1}^{jH}; V_{t+1}^{jH} - b^j \bar{L}\} + (1-\sigma^z)\max\{W_{t+1}^{jL}; V_{t+1}^{jL} - b^j \bar{L}\} \tag{14}$$

Notice that as $b^c = 0$ for an agent type cz at time t, $CV_{t+1}^{cz} = CW_{t+1}^{cz}$.

The consumption of an entrepreneur at time t is then equal to the net of wages earned, patronage received and net profits from the firm, after deducting the external debt service or repayment from the previous period.

Similarly, the value function of becoming a worker at time t for type jz is :

$$W_t^{jz} = w_t + P^j - D_{t-1}(1 + r^*) + \beta C W_{t+1}^{jz} \tag{15}$$

The consumption of a worker at time t is then equal to the net of wages earned and patronage received, after deducting the external debt service or repayment from the previous period.

Definition of economic equilibrium Given the sequence of tax policies p^{t*}, an economic equilibrium is defined as a sequence of agents decisions $x^{t*} = \{x_n\}_{n=1}^{\infty}$ and $k^{t*} = \{k_n\}_{n=1}^{\infty}$, a sequence of wages w^{t*} from time t onward, and entrepreneurship measure N_{Vt}^{m*} s.t:

- Given $p^{t*}$and w^{t*} and an agent's decisions, (x^{t*}, k^{t*}) solves the problem of the agents given by (11)
- All markets clear
- N_{Vt}^{m*} is consistent with the agents' individual decisions at any period t.

5.2.2 Solving for the economic equilibrium

Net value gain of Entrepreneurship Given the announced set of policies p^t, the occupational choice of agents at time t depends on their net value gain of becoming an entrepreneur at time t (NG). I derive the net value gain of entrepreneurship for an agent at time t conditional on their type as the following:

$$NG_t^m = V_t^m - W_t^m - \mathbb{I}_{\{s_t=r\}} b^j \bar{L} \tag{16}$$

Using (12), (15) and (16) the net gain of an entrepreneur of type $m = jzs$ at time t is given by the following expression:

$$NG_t^m = \frac{\alpha}{1-\alpha}(1-\tau_t^j)^{1/\alpha} A^z \bar{L} - w_t \bar{L} + \beta(1-\theta)(CV_{t+1}^{jz} - CW_{t+1}^{jz}) - \mathbb{I}_{\{s=r\}} b^j \bar{L} \tag{17}$$

Thus given the above expression for the net gain of entrepreneurship, an agent type m strictly prefers to become an entrepreneur at time t, their $NG_t^m > 0$ and be indifferent between becoming an entrepreneur or a worker if their $NG_t^m = 0$ and becomes a worker if $NG_t^m < 0$.

Equilibrium wages Notice from the net gain expression (17), the occupational decision and, therefore, the aggregate labour demand is a function of the wage. In what follows, I determine the equilibrium wages by deriving the aggregate labour demand and supply functions in the model. The aggregate supply function is constant at 1. The aggregate demand function depends on the measure of agents of each type that become entrepreneurs. To derive the labor demand function generated by each agent type m in equilibrium at time t, the first step is to derive the wage thresholds that make a given agent of type m indifferent between becoming an entrepreneur or staying a worker.

The value for the wage threshold for an agent type m at time t 'w_t^m' are derived by setting $NG_t^m = 0$. The individual becomes an entrepreneur if the equilibrium wage $w_t < w_t^m$, is indifferent if $w_t = w_t^m$ and becomes a worker if the equilibrium wage is above threshold. The threshold wages are as follows:

For a connected agent with skill z the threshold wage is:

$$w_t^{cz} = \frac{\alpha}{1-\alpha}(1-\tau_t^c)^{1/\alpha} A^z \tag{18}$$

For a non-connected worker with skill z:

$$w_t^{nzr} = \frac{\alpha}{1-\alpha}[(1-\tau_t^n)^{1/\alpha}A^z] - b^n + \frac{\beta(1-\theta)(CV_{t+1}^{nz} - CW_{t+1}^{nz})}{\bar{L}} \tag{19}$$

For a non-connected entrepreneur with skill z:

$$w_t^{nze} = \frac{\alpha}{1-\alpha}(1-\tau_t^n)^{1/\alpha}A^z + \frac{\beta(1-\theta)(CV_{t+1}^{nz} - CW_{t+1}^{nz})}{\bar{L}} \tag{20}$$

In what follows I derive relationships among these thresholds that will be useful in determining the aggregate labor demand curve.

Lemma 5.4. *The threshold wages satisfy the following inequalities:*

$$\begin{aligned} w_t^{cH} &> w_t^{cL} \\ w_t^{nHe} > w_t^{nHr} &> w_t^{nLr} \\ w_t^{nLe} &> w_t^{nLr} \end{aligned} \tag{21}$$

Proof. Based on the above wage thresholds (18)-(20) given that $A^H > A^L$ and $b^n > 0$, it is the case that wage thresholds are monotonically increasing in skill level z and decreasing in b^n, so that w_t^{cH} is always the highest and w_t^{nLr} is always the lowest and $w_t^{nze} > w_t^{nzr}$. $\square$

The ordering between w_t^{cL}, w_t^{nHr}, w_t^{nLe} depends on the policy sequence p^t. The next lemmas derive conditions that determine how these threshold wages are ordered.

Lemma 5.5. *Given* p^t *:*

(i) $w_t^{cL} \geq w_t^{nHe}$ *if and only if* $(1-\tau_t^n)^{1/\alpha}A^H + \frac{\beta(1-\theta)(CV_{t+1}^{nH} - CW_{t+1}^{nH})}{\bar{L}} \leq (1-\tau_t^c)^{1/\alpha}A^L$

(ii) $w_t^{cL} > w_t^{nHr}$ *if and only if* $\frac{\alpha}{1-\alpha}[(1-\tau_t^c)^{1/\alpha}A^L - ((1-\tau_t^n)^{1/\alpha}A^H + \frac{\beta(1-\theta)(CV_{t+1}^{nH} - CW_{t+1}^{nH})}{\bar{L}})] > -b^n$

(iii) $w_t^{nHr} > w_t^{nLe}$ *if and only if* $\frac{\alpha}{1-\alpha}(1-\tau_t^n)^{1/\alpha}(A^H - A^L) + \frac{\beta(1-\theta)((CV_{t+1}^{nH} - CW_{t+1}^{nH}) - (CV_{t+1}^{nL} - CW_{t+1}^{nL}))}{\bar{L}} > b^n$

Proof. Follows directly from the definitions of the wage thresholds. □

Based on Lemmas 5.4 to 5.5 five types of potential complete orderings of the wage thresholds are possible:

$$\textbf{Case 1:} \qquad w_t^{cH} > \boldsymbol{w_t^{cL} \geq w_t^{nHe} > w_t^{nHr} \geq w_t^{nLe}} > w_t^{nLr} \tag{22}$$

when p^t satisfy:

$$\frac{\alpha}{1-\alpha}[(1-\tau_t^c)^{1/\alpha}A^L - (1-\tau_t^n)^{1/\alpha}A^H - \frac{\beta(1-\theta)(CV_{t+1}^{nH} - CW_{t+1}^{nH})}{\bar{L}}] \geq 0 > -b^n$$
$$\& \quad \frac{\alpha}{1-\alpha}(1-\tau_t^n)^{1/\alpha}(A^H - A^L) + \frac{\beta(1-\theta)((CV_{t+1}^{nH} - CW_{t+1}^{nH}) - (CV_{t+1}^{nL} - CW_{t+1}^{nL}))}{\bar{L}} > b^n$$

$$\textbf{Case 2:} \qquad w_t^{cH} > \boldsymbol{w_t^{nHe} \geq w_t^{cL} > w_t^{nHr} \geq w_t^{nLe}} > w_t^{nLr} \tag{23}$$

when p^t satisfy:

$$0 \geq \frac{\alpha}{1-\alpha}[(1-\tau_t^c)^{1/\alpha}A^L - (1-\tau_t^n)^{1/\alpha}A^H - \frac{\beta(1-\theta)(CV_{t+1}^{nH} - CW_{t+1}^{nH})}{\bar{L}}] > -b^n$$
$$\& \quad \frac{\alpha}{1-\alpha}(1-\tau_t^n)^{1/\alpha}(A^H - A^L) + \frac{\beta(1-\theta)((CV_{t+1}^{nH} - CW_{t+1}^{nH}) - (CV_{t+1}^{nL} - CW_{t+1}^{nL}))}{\bar{L}} > b^n$$

$$\textbf{Case 3:} \qquad w_t^{cH} \geq w_t^{nHe} > \boldsymbol{w_t^{nHr} \geq w_t^{cL}} \geq w_t^{nLe} > w_t^{nLr} \tag{24}$$

when p^t satisfy:

$$\frac{\alpha}{1-\alpha}[(1-\tau_t^c)^{1/\alpha}A^L - (1-\tau_t^n)^{1/\alpha}A^H - \frac{\beta(1-\theta)(CV_{t+1}^{nH} - CW_{t+1}^{nH})}{\bar{L}}] \leq -b^n$$

Case 4:

$$w_t^{cH} > \boldsymbol{w_t^{cL}} > \boldsymbol{w_t^{nHe}} > \boldsymbol{w_t^{nLe}} \geq \boldsymbol{w_t^{nHr}} \geq w_t^{nLr} \tag{25}$$

when p^t satisfy:

$$\frac{\alpha}{1-\alpha}[(1-\tau_t^c)^{1/\alpha}A^L - (1-\tau_t^n)^{1/\alpha}A^H - \frac{\beta(1-\theta)(CV_{t+1}^{nH} - CW_{t+1}^{nH})}{\bar{L}}] \geq 0 > -b^n$$
$$\& \quad \frac{\alpha}{1-\alpha}(1-\tau_t^n)^{1/\alpha}(A^H - A^L) + \frac{\beta(1-\theta)((CV_{t+1}^{nH} - CW_{t+1}^{nH}) - (CV_{t+1}^{nL} - CW_{t+1}^{nL}))}{\bar{L}} \leq b^n$$

Case 5:

$$w_t^{cH} > \boldsymbol{w_t^{nHe}} > \boldsymbol{w_t^{cL}} > \boldsymbol{w_t^{nLe}} \geq \boldsymbol{w_t^{nHr}} \geq w_t^{nLr} \tag{26}$$

when p^t satisfy:

$$0 \geq \frac{\alpha}{1-\alpha}[(1-\tau_t^c)^{1/\alpha}A^L - (1-\tau_t^n)^{1/\alpha}A^H - \frac{\beta(1-\theta)(CV_{t+1}^{nH} - CW_{t+1}^{nH})}{\bar{L}}] > -b^n$$
$$\& \quad \frac{\alpha}{1-\alpha}(1-\tau_t^n)^{1/\alpha}(A^H - A^L) + \frac{\beta(1-\theta)((CV_{t+1}^{nH} - CW_{t+1}^{nH}) - (CV_{t+1}^{nL} - CW_{t+1}^{nL}))}{\bar{L}} \leq b^n$$

To analyze, in the paper, we introduce an additional assumption on parameters, which is satisfied in our calibration, that ensures that $V_t^{nL} - W_t^{nL} < 0 \ \forall \ t$, so that $w_t^{nHr} > w_t^{nLe}$ and eliminates cases 4 and 5. Section A.2 in the Appendix proves this formally.

Assumption 2. $\frac{\frac{\alpha}{1-\alpha}(1-\alpha)^{1/\alpha}(A^H - A^L)}{1-\beta(1-\theta)\sigma^H} > b^n$.

Figures 1-3 display the labour demand functions for cases 1-3 . Notice that labour demand is a decreasing step function of the wage rate. Each step shows the measure of total labour demanded by entrepreneurs of type m at time t, and the length of the step depends on the measure of agents of type m at time t. The equilibrium wage is at the point where labour demand is equal to 1, which, in turn, depends on assumptions on the initial distribution of agents.

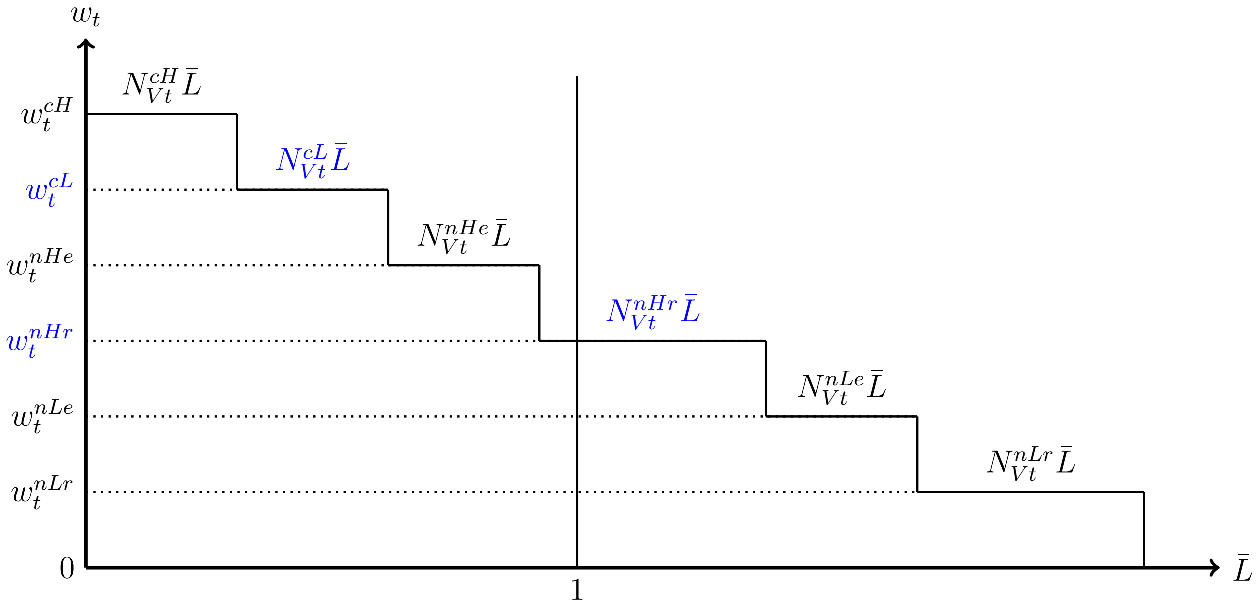


Figure 1: Labor demand in case 1

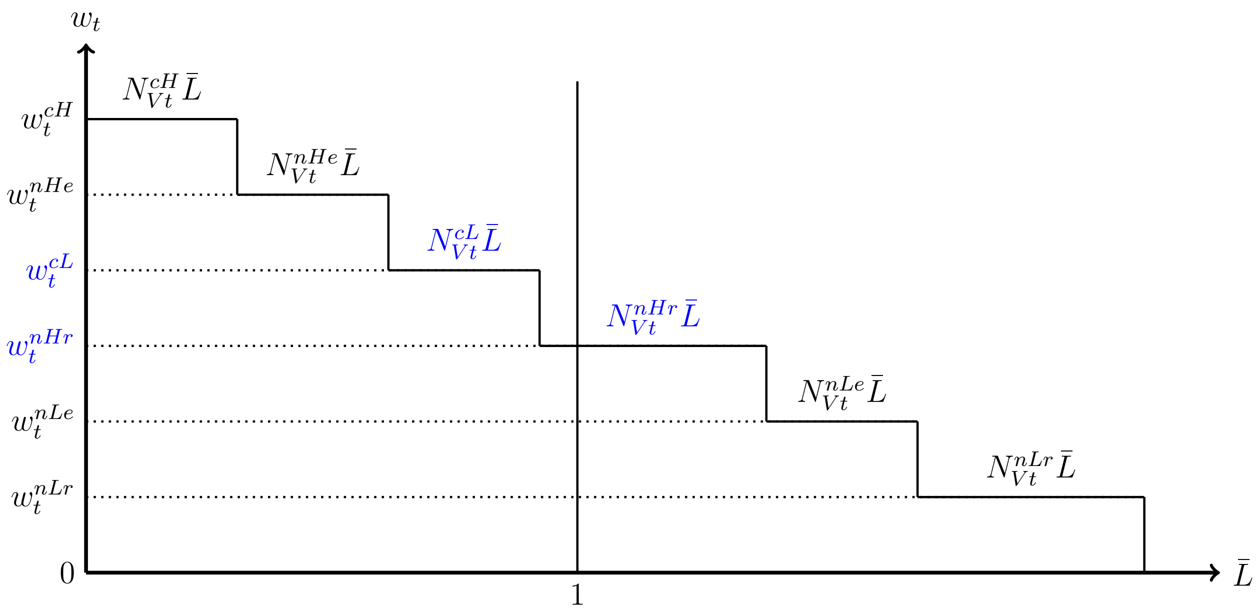


Figure 2: Labor demand in case 2

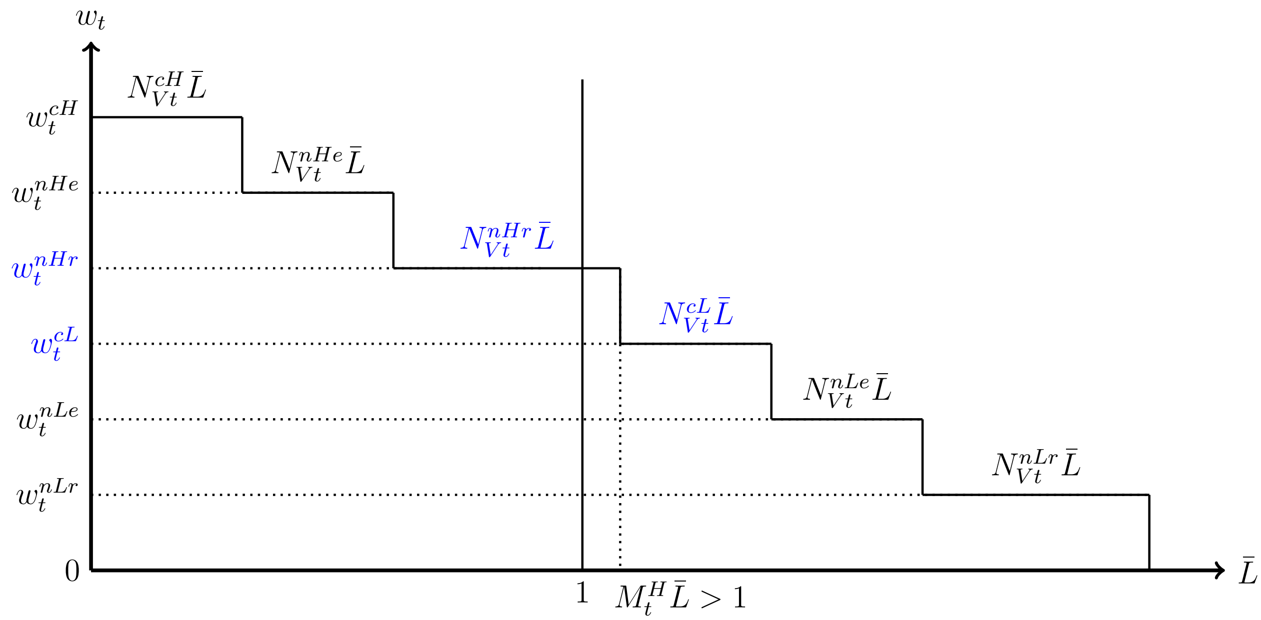


Figure 3: Labor demand in case 3

Lemma 5.6. *Under assumption 1, the equilibrium wage is* $w_t^* = w_t^{nHr}$ *for any policy sequence* p^t*, where* $\tau_t^n = \alpha\ \forall\ t$.

Proof. Using the measures of agents and the assumption 1 it can be shown that for:

Cases 1 and 2: All connected agents become entrepreneurs. Given assumption 1 (b) $\phi^c < \frac{1}{\bar{L}}$ it is the case that the total measure of connected are less than the total number of firms in equilibrium. Given assumption 1 (c) $M_t^H > \frac{1}{\bar{L}}$ the total number of connected high skilled agents plus the non-connected high skilled agents are greater than the total number of firms. Finally, given assumption 1(d) $N_0^e = 0$ and 1(f) $\phi^c + \sigma^H(1-\theta)(\frac{1}{\bar{L}} - \frac{1}{\bar{L}}\phi^c) < \frac{1}{\bar{L}}$ total number of non-connected existing entrepreneurs plus the total connected at any time t is less than the total number of equilibrium firms, this would make the equilibrium wage equal to w_t^{nHr} where, $NG_t^{nHr} = 0$ so that few but not all non-connected high skilled workers become entrepreneurs. Otherwise, if the wages are below w_t^{nHr} there will be excess demand of labor as all the non-connected high skilled agents would strictly prefer to become entrepreneurs and if they are above w_t^{nHr} there will an excess supply of labor as all non-connected high skilled non existing entrepreneurs would want to remain workers .

Cases 3: Given assumption 1 (c) $M^H > \frac{1}{\bar{L}}$ the total number of connected high skilled plus the non-connected high skilled agents are greater than the total number of firms. Finally, given assumption 1(d) $N_0^e = 0$ and 1(f) $\phi^c + \sigma^H(1-\theta)(\frac{1}{\bar{L}} - \frac{1}{\bar{L}}\phi^c) < \frac{1}{\bar{L}}$ and $(1-\sigma^H) > \sigma^L$ the total number of existing entrepreneurs plus the high skilled connected entrepreneurs at any time t is less than the total number of firms, this would make the equilibrium wage equal to w_t^{nHr} where, $NG_t^{nHr} = 0$ so that few but not all non-connected high skilled workers become entrepreneurs. Otherwise, if the wages are below w_t^{nHr} there will be excess demand of labour as all the non-connected high skilled agents would strictly prefer to become entrepreneurs and if they are above w_t^{nHr} there will an excess supply of labour as all non-connected high skilled non existing entrepreneurs would want to remain workers .

□

In what follows I am going to derive conditions based on the policy sequence p^t and the exogenous parameters of the model, to characterize the economic equilibrium.

Lemma 5.7. *Under lemma 5.6 and assumption 2 and given* $\tau_t^n = \alpha \ \forall \ t$, $\frac{\beta(CV_{t+1}^{nH} - CW_{t+1}^{nH})}{\bar{L}} = \beta\sigma^H b^n$.

Proof. Using that $w_t^* = w_t^{nHr}$ it is the case that $\forall t$, non-connected high skilled individuals who were not entrepreneurs in the previous period are indifferent between becoming a worker or an entrepreneur. Then in equilibrium NG_t^{nHr} =0, or $V_{t+1}^{nH} - b^n\bar{L} = W_{t+1}^{nH}$ so that non existing non-connected high skilled agents are indifferent between being an entrepreneur or a worker st time t. Using the fact that $V_{t+1}^{nH} > W_{t+1}^{nH}$ so that all high skilled non-connected existing entrepreneurs at time t always remain entrepreneurs. Assumption 2 implies that low skilled non-connected entrepreneurs will be always choose to remain workers so that $V_{t+1}^{nL} < W_{t+1}^{nL}$. Using these two : $CV_{t+1}^{nH} = \sigma^H V_{t+1}^{nH} + \sigma^{1-H} W_{t+1}^{nL}$ and $CW_{t+1}^{nH} = \sigma^H(V_{t+1}^{nH} - b^n\bar{L}) + \sigma^{1-H}W_{t+1}^{nL}$. Thus $CV_{t+1}^{nH} - CW_{t+1}^{nH} = \sigma^H b^n$. □

Notice from figures 1 to 3 that the labor supply is drawn as a constant at 1 and depicts the equilibrium wage $w_t^* = w_t^{nHr}$ for the cases 1, 2 and 3, where the total labor demanded is equal to the labor supply.

5.2.3 Characterization of Economic Equilibrium

Definition 1. *Let us define $\bar{\tau}$ as the threshold value of τ_t^c for which $w_t^{nHr} = w_t^{cL}$ as given by:*

$$\bar{\tau}^c = \left(\frac{A^L + b^n(1-\beta(1-\theta)\sigma^H)^{\frac{(1-\alpha)}{\alpha}} - (1-\alpha)^{\frac{1}{\alpha}} A^H}{A^L} \right)^{\alpha}.$$

Proposition 1. *Under assumptions 1, 2, given the policy (τ_t^c, α) there exists a unique economic equilibrium. In equilibrium $w_t^* = w_t^{nHr}$ $\forall t$ and the measures of entrepreneurs of each type at any time t take the form:*

1. ***(Type 1 (misallocation))**,*

   (a) $(N_{Vt}^{cH})^* = \sigma^L \phi^c + (\sigma^L - \sigma^H) N_{V_{t-1}}^{cH}$

   (b) $(N_{Vt}^{cL})^* = \phi^c - N_{Vt}^{cH}$

   (c) $(N_{Vt}^{nHe})^* = (1-\theta)\sigma^H N_{Vt-1}^{nH}$

   (d) $(N_{Vt}^{nHr})^* = (\frac{1}{L} - \phi^c) - N_{Vt}^{nHe}$

   (e) $(N_{Vt}^{nL})^* = 0$

   *If $\tau_t^c < \bar{\tau}$ **(condition 1)** OR*

2. ***(Type 2 (no misallocation))**,*

   (a) $(N_{Vt}^{cH})^* = \sigma^L \phi^c + (\sigma^L - \sigma^H) N_{V_{t-1}}^{cH}$

   (b) $(N_{Vt}^{cL})^* = 0$

   (c) $(N_{Vt}^{nHe})^* = (1-\theta)\sigma^H (N_{Vt-1}^{nH})$

   (d) $(N_{Vt}^{nHr})^* = (\frac{1}{L} - N_{Vt}^{cH}) - N_{Vt}^{nHe}$

   (e) $(N_{Vt}^{nL})^* = 0$

   *If $\tau_t^c \geq \bar{\tau}$ **(condition 2)**.*

Proof. The derivation of the condition (i) follows directly from comparing the wage thresholds w_t^{nHr} and w_t^{nLc} given in lemma 5.5 and the derivations in lemma 5.7. The measures of connected high skilled entrepreneurs are derived based on the transitional dynamics given in expression (3). The measure of existing and new non-connected high skilled entrepreneurs at period t depend on the measure and the Type of policy τ_{t-1}^c, that existed in period $t-1$. If the optimal τ_{t-1}^c satisfied condition 1 then $N_{Vt-1}^{nH} = (\frac{1}{L} - \phi^c)$, otherwise $N_{Vt-1}^{nH} = (\frac{1}{L} - \phi^c M_t^H)$. $\square$

This proposition establishes that, if τ_t^c satisfies **condition 1** at a given period t in equilibrium all connected (high and low skilled) agents are entrepreneurs causing misallocation. Out of the non-connected, all high skilled, who do not pay entry costs and some who do are entrepreneurs.

If τ_t^c satisfies **condition 2** in equilibrium all connected high skilled, all connected high skilled who do not pay the entry costs and a few non-connected high skilled who pay the entry costs are entrepreneurs. Therefore, only high skilled agents remain or become entrepreneurs leading to no resource misallocation in the economy.

Aggregate Resource Constraint

For simplicity let C_t denote the sum of the consumption of all connected and non-connected agents of different types at period t, where $C_t = \sum_m c_{Vt}^m N_{Vt}^m + \sum_m c_{Wt}^m N_{Wt}^m$ and K_t denote the capital invested by all the non-connected and connected entrepreneurs at the end of the period t, given by $K_t = \sum_m k_t^m N_{Vt}^m$. The aggregate resource constraint of the economy is then given by:

$$Y_t + D_t + F = \tilde{c}_t + B^n N_{Vt}^{nHr} + C_t + K_t + D_{t-1}(1 + r^*) \tag{27}$$

where $B^n N_{Vt}^{nHr}$ is the total expenditure on entry barriers by the entrepreneurs who have to pay the entry costs and Y_t is the total output in the economy produced by the connected and non-connected agents respectively at time t. Note that $D_{t-1} = D_t = D_{t+1} = D$ is fixed and constant over time.

5.2.4 Characterization of Stationary Economic Equilibrium

Lemma 5.8. *The sequence N^{cH}_{Vt} is monotone. In particular:*

(i) If $N^{cH}_{V0} < \phi^c M^H$, then N^{cH}_{Vt} is an increasing sequence converging to $\phi^c M^H$ in the stationary equilibrium.

(ii) If $N^{cH}_{V0} > \phi^c M^H$ then N^{cH}_{Vt} is a decreasing sequence converging to $\phi^c M^H$ in the stationary equilibrium.

Proof. It follows directly from assumption 1 and lemmas 5.1-5.2. □

Proposition 2. *Under assumptions 1, given policy $\tau^c_{t+1} = \tau^c_t = \tau^c$, there exists a unique stationary economic equilibrium. In equilibrium $w^*_{t+1} = w^*_t = w^{nHr}$ $\forall t$ and the measures of entrepreneurs of each type at any time t take the form:*

1. ***(Type 1 (misallocation))**,*

   (a) $(N^{cH}_V)^* = \phi^c M^H$

   (b) $(N^{cL}_V)^* = \phi^c(1 - M^H)$

   (c) $(N^{nHe}_V)^* = (1-\theta)\sigma^H(\frac{1}{L} - \phi^c)$

   (d) $(N^{nHr}_V)^* = (1-(1-\theta)\sigma^H)(\frac{1}{L} - \phi^c)$

   (e) $(N^{nL}_V)^* = 0$

   *If $\tau^c < \bar{\tau}$ **(condition 1s)** OR*

2. ***(Type 2 (no misallocation))**,*

   (a) $(N^{cH}_V)^* = \phi^c M^H$

   (b) $(N^{cL}_V)^* = 0$

(c) $(N_V^{nHe})^* = (1-\theta)\sigma^H(\frac{1}{L} - \phi^c M^H)$

(d) $(N_V^{nHr})^* = (1-(1-\theta)\sigma^H)(\frac{1}{L} - \phi^c M^H)$

(e) $(N_V^{nL})^* = 0$

If $\tau^c \geq \bar{\tau}$ ***(condition 2s)****.*

This proposition establishes that, if τ^c satisfies **condition 1** in stationary economic equilibrium all connected (high and low skilled) agents are entrepreneurs causing misallocation in the long run. Out of the non-connected, all high skilled, who do not pay entry costs and some who do are entrepreneurs.

If τ^c satisfies **condition 2** in a stationary economic equilibrium all connected high skilled, all non-connected high skilled who do not pay the entry costs and a few non-connected high skilled who pay the entry costs are entrepreneurs. Therefore, only high skilled agents remain or become entrepreneurs leading to no long run resource misallocation in the economy. Notice that I have assumed that if the condition 1 is satisfied with equality, high skilled agents enter first and the economy is in Type 2 equilibrium, since $M_t^H < \frac{1}{L}\ \forall t$. Also notice that the total number of non-connected high skilled entrepreneurs is higher in Type 2 compared to a Type 1 equilibrium with misallocation.

5.2.5 Political Equilibrium

Elite's problem Given the sequence of announced policies $\hat{p^t}$ the Elite maximizes their preferences taking into account the occupational decisions by agents determined in the economic equilibrium. Using the notation discussed in subsection 5.1.2, the Elite's problem is as follows:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (1-\delta(\tau_t^c))\tilde{c}_t$$

Subject to:

$$(T_t^n)(\tau_t^c) + (T_t^c)(\tau_t^c) + F + D - P^c\phi^c \geq (\tilde{c}_t)$$

$$\tau_t^n = \alpha$$

Let $N_{Vt}^{cH} + N_{Vt}^{cL}(\tau^c)$ be the measure of total connected entrepreneurs at time t. Where:

1. $T_t^c(\tau_t^c) = \frac{1}{1-\alpha}\tau_t^c(1-\tau_t^c)^{\frac{1-\alpha}{\alpha}}\bar{L}[N_{Vt}^{cH}A^H + (N_{Vt}^{cL})(\tau_t^c)A^L]$is the total tax revenue from the connected entrepreneurs.

2. $T_t^n(\tau_t^c) = \frac{1}{1-\alpha}\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}\bar{L}[\frac{1}{\bar{L}} - (N_{Vt}^{cH} + N_{Vt}^{cL}(\tau^c))A^H$ is the total tax revenue from the non-connected entrepreneurs.

Given assumption 1, there are two things to notice from the above expressions. First, that the level of tax revenues from the non-connected entrepreneurs does not depend on their type s at time t, it only depends on the total measure of non-connected high skilled entrepreneurs. Second, due to the two types of economic equilibria, the problem is not differentiable and there is a discontinuity on the agent's decisions when τ_t^c satisfies condition 2 with equality.

The next lemma shows that tax revenues at period t depend only on the period tax rate τ_t^c and the (exogenously determined) measure of high skilled entrepreneurs M_t^H, and not on previous period decisions.

Lemma 5.9. *Assume assumption 1 holds. Given $\tau_t^n = \alpha$ and M_0^H, T_t^c and T_t^n at time t are functions of τ_t^c and M_t^H.*

Proof. It follows from proposition 1 :

1. If condition 1 is satisfied the measure of total connected agents is $N_{Vt}^{cH} + N_{Vt}^{cL} = \phi^c \ \forall t$. Therefore, the total measure of non-connected high skilled is $N_{Vt}^{nH} = \frac{1}{\bar{L}} - \phi^c$ and is a constant.

2. If condition 2 is satisfied the measure of total connected agents is $N_{Vt}^{cH} = \phi^c M_t^H$ and $N_{Vt}^{cL} = 0 \ \forall t$. Therefore, the measure of non-connected high skilled is $N_{Vt}^{nH} = \frac{1}{\bar{L}} - \phi^c M_t^H$.

Therefore, the tax revenues in period t if condition (1) is satisfied can be written as:

1. $(T_t^c)^1(\tau_t^c) = \frac{1}{1-\alpha}\tau_t^c(1-\tau_t^c)^{\frac{1-\alpha}{\alpha}}\bar{L}[\phi^c M_t^H A^H + \phi^c(1-M_t^H)A^L]$ from the connected entrepreneurs.

2. $(T_t^n)^1(\tau_t^c) = \frac{1}{1-\alpha}\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}\bar{L}(\frac{1}{\bar{L}} - \phi^c)A^H$ from the non-connected entrepreneurs.

The tax revenues in period t if condition (2) is satisfied can be written as:

1. $(T_t^c)^2(\tau_t^c) = \frac{1}{1-\alpha}\tau_t^c(1-\tau_t^c)^{\frac{1-\alpha}{\alpha}}\bar{L}[\phi^c M_t^H A^H]$ from the connected entrepreneurs.

2. $(T_t^n)^2(\tau_t^c) = \frac{1}{1-\alpha}\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}\bar{L}(\frac{1}{\bar{L}} - \phi^c M_t^H)A^H$ from the non-connected entrepreneurs.

□

Thus, given lemma 5.9 the Elite's budget constraint in period t depends only on period t variables. Therefore, the problem of the Elite can be reduced to solving an infinite set of static problems, one per period. I now separate the problem in two parts: Solve for $\tilde{u_t}^1$ the maximum period utility subject to the Elite's budget constraint if condition 1 is satisfied at period t and $\tilde{u_t}^2$ the maximum period utility subject to the Elite's budget constraint if condition 2 is satisfied at period t. The Elite's lifetime utility function can then be expressed as:

$$\tilde{U}_t = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \max\{\tilde{u_t}^1, \tilde{u_t}^2\} \tag{28}$$

The optimal policy (τ_t^c, α) and the type of economic equilibrium at period t will then be determined by comparing the two utilities $\tilde{u_t}^1$ and $\tilde{u_t}^2$ and choosing the one that maximizes the Elite's period utility as given by equation (28). Since in equilibrium $\tau_t^j = \hat{\tau}_t^j$, to economize on notation I omit the conditional expected policies notation $(\hat{\tau}_t^c, \alpha)$.

Let the net total revenues for the Elite in an economic equilibrium Type i at time t expressed as:

$$(TR_t)^i(\tau_t^c) = (T_t^c)^i(\tau_t^c) + (T_t^n)(\tau_t^c)^i + F + D + P^c\phi^c$$

where $(TR_t)^i(\tau_t^c)$ are the total revenues from taxes on entrepreneurs if condition$i$ is satisfied, D and F are the external debt and foreign aid inflows in the economy.

The constraint on the net total revenues is then given by $(TR_t)^i(\tau_t^c) \geq 0$. Therefore, the Elite need to raise at least revenue $P^c\phi^c$. Notice that $(T_t^c)^i(\tau_t^c) + (T_t^n)^i(\tau_t^c) > 0$ regardless of the type of economic equilibrium. Henceforth for simplicity, I assume that for the economies where the connections prevail $D + F + T_t^c + T_t^n$ is large and $P^c\phi^c$ is small which is more near to the real world developing economies.

The Elite's optimization problem assuming it chooses $(\tau_t^c, \tau_t^n = \alpha)$ satisfying (condition i') :

$$\tilde{u}_t{}^i = \max_{\tau_t^c}(1 - \delta(\tau_t^c))\tilde{c}_t{}^i \tag{29}$$

Subject to:

$$(T_t^c)^i(\tau_t^c) + (T_t^n)(\tau_t^c)^i + F + D - P^c\phi^c \geq \tilde{c}_t{}^i$$

$$\tau_t^c \ satisfies \ condition \ i'$$

Where condition 1' is equal to condition 1 with a weak inequality and condition 2' is equal to condition 2.

I now derive the expression for τ_t^c for which the condition (i) is binding with equality.

Proposition 3. *If **condition i** is binding at time t, then $(\tau_t^c)^i$ the solution to (29) for type i economic equilibrium is given by $\bar{\tau}$.*

Proof. Given that $\tau_t^n = \alpha$. The value of $(\bar{\tau}^c)$ derives from **condition i** being satisfied with equality. □

In what follows, I solve for $(\tau_t^c)^1$ and $(\tau_t^c)^2$, that maximizes $\tilde{u}_t{}^1$ and $\tilde{u}_t{}^2$ without the economic constraint given by condition 1' and 2, assuming that the distributions of agents are such that conditions 1' and 2 are satisfied respectively. Next, I verify whether the unconstrained solution satisfies condition 1' and 2. If it does then the solution to the unconstrained problem (29) is given by $(\tau_t^c)^i$. If it does not then $(\tau_t^c)^i = \bar{\tau}_t^c$. I then discuss potential cases where $\tilde{u_t}^2$ might be greater or lower than $\tilde{u_t}^1$ at a given period t. Also notice that at this point it is not feasible to derive any additional properties of the potential equilibrium $(\tau_t^c)^*$ without having special values of the parameters and a specific functional form for $\delta(\tau_t^c)$.

5.2.6 Solving for Political Equilibrium

Type 1 Economic Equilibrium Assume that the Elite chooses a τ_t^c that satisfies condition 1, then the economic equilibrium will be of type 1, and the distribution of entrepreneurs will be given by Type 1 in proposition 1. The Elite will solve the problem:

$$\tilde{u_t}^1 = \max(1 - \delta(\tau_t^c))\tilde{c_t}^1 \tag{30}$$

Subject to:

$$(T_t^c)^1(\tau_t^c) + (T_t^n)^1(\tau_t^c) + F + D - P^c\phi^c \geq (\tilde{c}_t)^1$$

$$\frac{\alpha}{1-\alpha}[(1-\tau_t^c)^{1/\alpha}A^L - (1-\alpha)^{1/\alpha}A^H] \geq b^n(\beta(1-\theta)\sigma^H - 1)$$

Let us solve the problem by guess and verify: solve the problem assuming a type 1 economic equilibrium at time t and ignoring **condition 1** and then check whether the solution satisfies **condition 1**.

Step 1: Solve for the unconstrained Elite's problem given in (30).

The First order condition for the unconstrained problem after substitution for consumption $\tilde{c_t}^1$ in (30) is given by:

FOC wrt (τ_t^c)**:**

$$[(1-\delta(\tau_t^c))(T_t^{c'})^1(\tau_t^c) - \delta'(\tau_t^c)(TR_t)^1(\tau_t^c)] = 0 \tag{31}$$

Where $(T_t^{c'})^1(\tau_t^c) = \bar{L}(\phi^c M_t^H A^H + \phi^c(1-M_t^H)A^L)(1-\tau_t^c)^{\frac{1-\alpha}{\alpha}}(\frac{\alpha-\tau_t^c}{\alpha(1-\alpha)(1-\tau_t^c)})$ and $\delta'(\tau_t^c) > 0$.

Step 2: Check if the interior solution $(\tau_t^c)^1$ given by (31) for the unconstrained problem satisfies **condition 1**. If $(\tau_t^c)^1$ given by (31) does not satisfy **condition 1** then $(\tau_t^c)^1 = \bar{\tau}_t^c$ at time t .

Type 2 Economic Equilibrium Assume that the Elite chooses a τ_t^c that satisfies condition 2, then the economic equilibrium will be of type 2, and the distribution of entrepreneurs will be given by Type 2 in proposition 1. The Elite will solve the problem:

$$\tilde{u_t}^2 = \max(1-\delta(\tau_t^c))\tilde{c_t}^2 \tag{32}$$

Subject to:

$$(T_t^c)^2(\tau_t^c) + (T_t^n)^2(\tau_t^c) + F + D - P^c\phi^c \geq (\tilde{c}_t)^2$$

$$\frac{\alpha}{1-\alpha}[(1-\tau_t^c)^{1/\alpha}A^L - (1-\alpha)^{1/\alpha}A^H] \leq b^n(\beta(1-\theta)\sigma^H - 1)$$

Let us solve the problem by guess and verify: solve the problem assuming a type 2 economic equilibrium at time t and ignoring **condition 2** and then check whether the solution satisfies **condition 2**.

Step 1: Solve for the unconstrained Elite's problem given in (32).

The First order condition for the unconstrained problem after substitution for consumption $\tilde{c_t}^2$ in (32) are given by:

FOC wrt $(\tau_t^c)^2$**:**

$$[(1-\delta(\tau_t^c))(T_t^{c'})^2(\tau_t^c) - \delta'(\tau_t^c)(TR_t)^2(\tau_t^c)] = 0 \tag{33}$$

Where $(T_t^{c'})^2(\tau_t^c) = \bar{L}(\phi^c M_t^H A^H)(1-\tau_t^c)^{\frac{1-\alpha}{\alpha}}(\frac{\alpha-\tau_t^c}{\alpha(1-\alpha)(1-\tau_t^c)})$ and $\delta'(\tau_t^c) > 0$.

Step 2: Check if the interior solution $(\tau_t^c)^2$ given by (31) for the unconstrained problem satisfies **condition 1**. If $(\tau_t^c)^2$ given by (31) does not satisfy **condition 2** then $(\tau_t^c)^2 = \bar{\tau}_t^c$ at time t .

The following lemmas discuss some properties of the potential solutions to $\tilde{u_t}^1$ and $\tilde{u_t}^2$ at period t.

Lemma 5.10. *If the interior solution $(\tau_t^c)^1$ given by (31) does not satisfy* ***condition 1*** *and the interior solution $(\tau_t^c)^2$ given by (33) does not satisfy* ***condition 2*** *then $(\tau_t^c)^1 = (\tau_t^c)^2 = \bar{\tau}_t^c$.*

Proof. Directly follows from **condition 1** and **condition 2**. □

Lemma 5.11. *Assume assumption 1 holds. If $(\tau_t^c)^2$ is an interior solution then $(\tau_t^c)^1 = \bar{\tau}^c$.*

Proof. By contradiction. Assume that $(\tau_t^c)^1$ is interior. It follows from lemma 5.9 that $(TR_t)^2(\tau_t^c)^1 > (TR_t)^1(\tau_t^c)^1$ and $(T_t^c)^{2'}(\tau_t^c)^1 < (T_t^c)^{1'}(\tau_t^c)^1$. Notice from the first order conditions given by (31) and (33) that $(1-\delta(\tau_t^c)) > 0$ and decreasing in τ_t^c, $(T_t^c)'(\tau_t^c) > 0$ and is decreasing for $\tau_t^c \in (0, \alpha)$ and $\delta'(\tau_t^c) > 0$. Therefore, at the given $(\tau_t^c)^1$ the FOC for Type 2 economic equilibrium given by (33) is negative, as the connected measures of entrepreneurs are lower and the non-connected higher, paying higher taxes. Thus, the unconstrained interior solution that satisfies (33), will be lower than $(\tau_t^c)^1$ such that $(\tau_t^c)^2 < (\tau_t^c)^1$. Therefore, $(\tau_t^c)^2 < (\bar{\tau})$, and does not satisfy condition 2. A contradiction. □

Given the above possible solutions, in the following proposition I discuss all potential cases and possible solutions as to the type of economic equilibrium that maximizes the Elite's period utility $\tilde{u}_t$ for a given period t.

Proposition 4. *Assume assumption 1 holds. Then it is the case that in any given period t:*

1. *If $(\tau_t^c)^1 = (\tau_t^c)^2 = \bar{\tau}^c$, then ${\tilde{u}_t}^2 > {\tilde{u}_t}^1$*
2. *If $(\tau_t^c)^1 = \bar{\tau}^c$ and $(\tau_t^c)^2$ is an interior solution, then ${\tilde{u}_t}^2 > {\tilde{u}_t}^1$*
3. *If $(\tau_t^c)^2 = \bar{\tau}^c$ and $(\tau_t^c)^1$ is an interior solution, then ${\tilde{u}_t}^2 > {\tilde{u}_t}^1$ or ${\tilde{u}_t}^2 < {\tilde{u}_t}^1$*

Proof.

1. If $(\tau_t^c)^1 = (\tau_t^c)^2 = \bar{\tau}_t^c$, $\delta(\bar{\tau}_t^c)^1 = \delta(\bar{\tau}_t^c)^2$. It follows from lemma 5.9 that $(TR_t)^2(\bar{\tau}_t^c) > (TR_t)^1(\bar{\tau}_t^c)$. Thus, ${\tilde{u}_t}^2 > {\tilde{u}_t}^1$. The economic equilibrium at period t is of Type 2 and the optimal equilibrium policy $(\tau_t^c)^* = (\bar{\tau}^c)$.
2. The proof follows from lemma 5.11 and lemma 5.9. Given the previous case, the interior solution that maximizes the period utility from being in Type 2 economic equilibrium at period t ${\tilde{u}_t}^2(\tau_t^c)^2$ is always higher than the utility ${\tilde{u}_t}^2(\bar{\tau}^c)$ when condition 2 is binding. Thus, if ${\tilde{u}_t}^2 > {\tilde{u}_t}^1$ when $(\tau_t^c)^1 = (\tau_t^c)^2 = \bar{\tau}_t^c$ then it is always the case that ${\tilde{u}_t}^2 > {\tilde{u}_t}^1$, when $(\tau_t^c)^1 = \bar{\tau}^c$ and $(\tau_t^c)^2$ is an interior solution. The economic equilibrium at period t is of Type 2 and the optimal equilibrium policy $(\tau_t^c)^* = (\tau_t^c)^2$.
3. Depending on the parameters of the model that determine $\bar{\tau}^c$, the external flows and the functional form for $\delta(\tau_t^c)$, if $(\tau_t^c)^2 = \bar{\tau}^c$ and $(\tau_t^c)^1$ is an interior solution. Then the Elite's utility under Type 1 and Type 2 economic equilibrium at time t will be compared and which ever is higher will determine the Elite's period utility maximizing policy $(\tau_t^c)^*$. If ${\tilde{u}_t}^2 > {\tilde{u}_t}^1$, then $(\tau_t^c)^* = \bar{\tau}^c$ and if ${\tilde{u}_t}^2 < {\tilde{u}_t}^1$, then $(\tau_t^c)^* = (\tau_t^c)^1$.

□

Stationary Equilibrium A stationary equilibrium will then consist of an equilibrium where the measures of the entrepreneurs of each type N_V^m do not change over time and the optimal policy $(\tau^c_{t+1})^* = (\tau^c_t)^* = (\tau^c)^*\ \forall t$. Notice that because the Elite's period problem was solved as a static problem per period, the stationary problem will also be solved following the procedure discussed above, with the stationary distribution of entrepreneurs given in proposition 2. The following propositions and lemma summarize the potential Type of economic equilibrium and the optimal $(\tau^c)^*$ that can exist under different conditions in the steady state.

Proposition 5. *Assume assumption 1 holds. Then it is the case that in steady state:*

1. *If $(\tau^c)^1 = (\tau^c)^2 = \bar{\tau}^c$, then $\tilde{u}^2 > \tilde{u}^1$*
2. *If $(\tau^c)^1 = \bar{\tau}^c$ and $(\tau^c)^2$ is an interior solution, then $\tilde{u}^2 > \tilde{u}^1$*
3. *If $(\tau^c)^2 = \bar{\tau}^c$ and $(\tau^c)^1$ is an interior solution, then $\tilde{u}^2 > \tilde{u}^1$ or $\tilde{u}^2 < \tilde{u}^1$*

Proof. Arguments similar to the proof for proposition 4. □

Proposition 6. *Given assumption 1 and 2 there exists a stationary equilibrium of Type 1 if $(\tau^c)^* = (\tau^c)^1$ satisfy condition 1 and the measures of the entrepreneurs of each type N^m_V are given by proposition 2 for a Type 1 stationary economic equilibrium. There exists a stationary equilibrium of Type 2 if $(\tau^c)^* = (\tau^c)^2$ satisfy condition 2 and the measures of the entrepreneurs of each type N^m_V are given by proposition 2 for a Type 2 stationary economic equilibrium.*

Proof. Follows directly from proposition 5 so that if $\tilde{u}^1 > \tilde{u}^2$ then $(\tau^c)^* = (\tau^c)^1$ and vice versa. □

The following lemma shows that if the Elite's per period utility is maximized from being in a Type 1 economic equilibrium in the initial and the steady state period, the economy will be in Type 1 along the whole equilibrium path.

Lemma 5.12. *It is the case that $(\tau^c_t)^* = (\tau^c_t)^1\ \forall t$, where $(\tau^c_t)^1$ is an interior solution $\forall t$ if and only if $u^1_0 > u^2_0$ and $\tilde{u}^1 > \tilde{u}^2$.*

Proof. Given that the initial measure of entrepreneurs is zero and the measure of entrepreneurs is a function of M_t^H. The monotonicity and convergence follows from lemma 5.8. $\square$

Analysis: The role of the net external flows

Notice that if case 3 in **proposition 4** hold then the level of external inflows $D+F$ is crucial in determining the Type of economic equilibrium in the economy. If the total revenues from the net external flows increase then the total net revenues for the Elite increase and they are less dependent on revenues from taxes to maintain their consumption. Notice from the FOC's given by (31) and (33), if condition i is not binding and $(\tau_t^c)^i$ is an interior solution any exogenous increase in $(TR_t)^i(\tau_t^c)$ at the existing policy make the FOC's negative and a lower $(\tau_t^c)^i$ will be needed to satisfy the FOC's.

As in case 3 of **proposition 4**, if **condition 2** binds then optimal $(\tau_t^c)^*$ in Type 2 equilibrium is a corner solution. Therefore, any increase in external flows do not change the optimal policy $(\tau_t^c)^2$ for Type 2 . However, the optimal policy for Type 1 $(\tau_t^c)^1$ is an interior solution and any increase in $(TR_t)^1(\tau_t^c)$ will decrease $(\tau_t^c)^1$ to satisfy first order condition given by equation (31). Thus the effect on Elite's utility $\tilde{u_t}^1$ is two fold. First, the Elite's consumption increases from the increase in the external flows. Second, there is an increase in the Elite's probability to remain in power as $\delta(\tau_t^c)$ decreases. Where as, the increase in the Elite's utility $\tilde{u_t}^2$ under Type 2 economic equilibrium only occurs due to the increase in its consumption from the extra external flows, there is no change in $\delta(\tau_t^c)$.

Intuitively, if external flows are high enough this decreases the Elite's dependence on tax revenues. Thus, the elite is able to lower the tax rate on the connected to increase their ability to remain in power and if $\tau_t^c < \bar{\tau}$, this results in a Type 1 economic equilibrium with misallocation and a measure of low skilled entrepreneurs entering and producing in the economy. However, when the external flows decrease the Elite's dependence on the tax revenues increases. As the tax rate for the non-connected is already at the maximum level, they can only raise tax revenues by either having larger fraction of non-connected entrepreneurs or increasing the tax rate for the connected. Both of which collectively leads to a Type 2 economic equilibrium with no misallocation. Therefore, higher net external flows is more likely to perpetuate an economic equilibrium with misallocation, as they lower the $(\tau_t^c)^1$ leading to a greater increase in Elite's utility under Type 1 economic equilibrium compared to Type 2 economic equilibrium with no misallocation.

6 Calibration

In this section I calibrate the parameters of the model developed in section 5, which are reported in table 2.

Table 2: Parameters

Name of the Variable (notation)	**Value**	**Source**
Discount factor (β)	0.99	Standard value in literature
Labor share of output (α)	0.42	Pakistan's labor share of income 2018[1]
High skilled ability (A^H)	2.61	Pakistan's skill premium ratio 2018[1]
Low skilled ability(A^L)	1	Normalized to 1
Probability of a high skilled to high skilled (σ^H)	0.74	Pakistan's 1-downward mobility 2012-13[2]
Probability of a low skilled to high skilled (σ^L)	0.18	Pakistan's Upward mobility 2012-13[2]
Measure of connected agents (ϕ^c)	0.30	Average measure of connected firms PSX in 2013-2019[3]
Fraction of firms with exit shock (θ)	0.7	$\phi^c + (1-\theta)\sigma^H(\frac{1}{\bar{L}} - \phi^c \frac{1}{\bar{L}}) < \frac{1}{\bar{L}}$
Calibration		
Name of the Variable (notation)	Value	Target Statistics
Net external debt inflow (D)	0.06	2.9% of the GDP[4]
Foreign aid flow (F)	0.04	2% of the GDP[5]
World interest rate (r^*)	0.02	Pakistan's average interest rate on new external debt commitments[6]
Total Patronage transfer ($P^c\phi^c$)	0.18	9% of the GDP[2]
Firm size (labor employed) ($\bar{L}$)	2.8	% of firms filed income tax with middle to high income 2017-2018[7]
Fixed cost of entry (B^n)	0.76	10% of the lifetime output of a existing non-connected[8] high productivity firm
δ parameters		
(a)	25	CPI as a proxy for the slope[9]
(b)	0.3	Factor share of labour[2]

[1]UNDP (2020), [2]Muhammad & Jamil (2017), [3]Imran (2023),[4]World Bank, IDS (2020),[5]OECD, CRS (2020), [6]Pakistan Economic Survey (2020),[7]Federal Board of Revenue, Pakistan (FBR) (2017-18), [8]Afraz et al (2014), [9]Corruption Perception Index, Transparency international (2020).

The discount factor β is set to a standard value of 0.99. The labour share of income α is set to 0.42, following Pakistan's labour share of income in 2018 as measured by the UNDP (2020). The high skilled ability A^H is set to 2.61 and this is based on the skill premium ratio calculated as the ratio between the wages of two most high skilled and two most low skilled categories of workers, obtained from UNDP (2020) report. As the skill premium is a ratio I normalize the ability of low skilled entrepreneurs to be $A^L = 1$. The measure of connected agents ϕ is set to 0.3 so that the average percentage of total politically connected firms in a steady state equilibrium is 60%, which is equivalent to approximately the average number of political connected listed Pakistani firms for the period of 2013-2019 as reported in table 6 in the Appendix.[6]. The probability of staying high skilled σ^H and transitioning from low to high skilled σ^L are set to match one minus the estimated ratio of the mobility of labour from high to low skilled and the mobility of labour from low to high skill workers in Pakistan respectively (Muhammad & Jamil, 2017).

No reliable data exists on estimating the costs of bribes paid or bureaucratic procedures faced by firms in Pakistan. There are several studies and papers documenting high barriers constituting of bureaucratic procedures, access to credit, lack of access to electricity, corruption and political instability for firms. In a study on the barriers to growth and entry for small and medium enterprises in Pakistan, Afraz et al. (2014) find that lack of access to utilities such as electricity and gas and bribes paid to obtain these facilities alone can result in loss of more than 10% of the total annual sales of a firm. In addition, bribes paid to the government officials might constitute up to an additional 4.2 percent of the total sales contract of a firm. Note that in the model the entry cost is only paid once by the new firms when they enter and these firms will not pay the entry barriers again if they do not exit entrepreneurship in their lifetime. Thus, I calibrate the fixed cost of entry B^n to match a conservative estimate of 10%

[6] The average percentage of firms that are connected in a steady state equilibrium is calculated by taking the average of the politically connected firms in Type 1 and Type 2 steady state as a percentage of the total firms in equilibrium

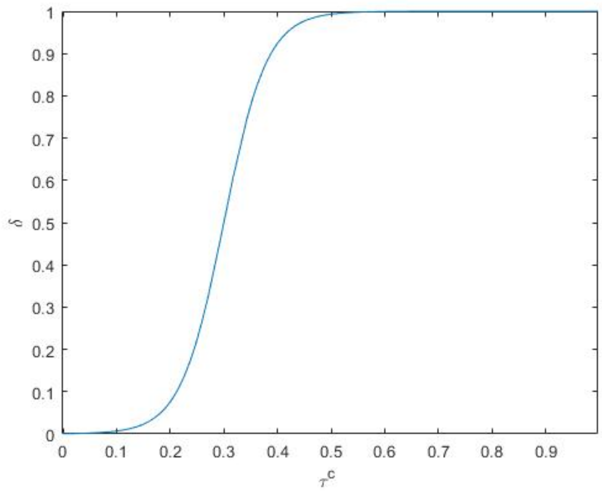


Figure 4: Functional form for $\delta(\tau_t^c)$ for $a = 25$ and $b = 0.3$

of the annual lifetime before tax output of a non-connected firm conditional on remaining an entrepreneur in the model, which results in the calibrated per worker entry cost b^n of 0.27.

The model requires a functional form for delta which is increasing in τ_t^c. I consider the following functional form for $\delta(\tau_t^c)$:

$$\delta(\tau_t^c) = \frac{1}{1 + e^{-a*(\tau_t^c - b)}} \tag{34}$$

I calibrate $a = 25$ and $b = 0.3$. Figure 4 shows it graphically. The slope of the function is controlled by the coefficient a. This slope determines how strongly the Elite's ability to stay in power depends on the privileges given to the connected in terms of lower taxes. Where as, given the slope a the coefficient b determines the position of $\delta(\tau_t^c)$ and influence the level of τ_t^c for which the function $\delta(\tau_t^c)$ reaches 1. The coefficient a and b collectively are chosen to match the factor labour share for Pakistan $\alpha = 0.42$, this represents a point where the values of $\delta(\tau_t^c)$ goes closer to 1. The calibrated parameters for coefficients a and b are also consistent with the following (i) In the model $\delta(\tau_t^c) = 1$ for $\tau_t^c > \tau_t^n$, where $\tau_t^n = \alpha = 0.42$

is the tax rate imposed on non-connected in equilibrium. [7]and (ii) Pakistan's corruption is ranked twice as that of South Korea, a country known for it's rapid transformation to a developed economy, as measured by the Corruption perception index (CPI) (Transparency International, 2018). Using the CPI as the proxy for coefficient a, 25 is also the minimum number that if reduced by half, the baseline model results in the equilibrium with lower misallocation. Given other parameters any coefficient bigger than 12.5 would reciprocate Pakistan being in an equilibrium with higher misallocation. Sensitivity analysis is also performed by changing on the value of coefficient a.

Based on the facts related to Pakistan's environment discussed in section 3, the rest of the exogenous parameters of the model are calibrated to a steady state period of a Type 1 equilibrium specifications. The firm size $\bar{L}$ is set to match the percentage of registered business who filled income taxes in Pakistan with middle to high income for the year 2017-2018 as reported by the Federal Board of Revenue, Pakistan. I back out $\bar{L}$ by setting the percentage of non-connected high skilled firms to be 16% of the total firms in the stationary equilibrium of Type 1. The fraction of entrepreneurs getting an exogenous exit shock θ is then set to 0.7 for which the assumption $\phi^c + (1-\theta)\sigma^H(\frac{1}{L} - \frac{1}{L}\phi^c) < \frac{1}{L}$ on the distribution is close to binding.[8] There exists no statistical data regarding the firm level entry and exit rate for Pakistan, the value of 0.7 corresponds to the entry rate of new firms in a Type 1 equilibrium equivalent to 12.5%, which is close to the average yearly entry rate of new firms in US for the past 10-15 years (Orazem & Winters, 2023).

The external debt inflow and foreign aid D and F are calibrated to match Pakistan's average current account deficit to GDP ratio and the average foreign aid to GDP ratio for the period of (1976-2021) which are taken from World Bank (2020) and Oecd (2018). Current account

[7]Pakistan's average corporate tax rate is 33% with 43% being the highest and 29% being the lowest in the last two decades

[8]Substituting in the values for the parameters in $\phi^c + (1-\phi^c)\sigma^e < \frac{1}{L} = 0.3 + (0.3)(0.74)(0.3 - 0.108) = 0.342 < 0.357$

deficit indicates a positive net capital inflows for the country. Recently, the current account deficit has increased rapidly and it reached an all time high at 6.6% of GDP in 2018 and the net foreign direct inflows for Pakistan on average are declining and make less than 0.5% of Pakistan's GDP (World Bank, 2020).

Patronage amount P^c is calibrated to match the ratio of total benefit of the government expenditure as a percentage of GDP, accrued to the richest quintile in 2018. This also constitutes the most politically connected group and is obtained from UNDP (2020).

7 Baseline Model Simulation

In this section the model is solved numerically for the values of parameters calibrated above. The subsection 7.1 presents the steady state simulation results of the baseline model and is followed by subsection 7.2 which analyses the equilibrium path for the policy and other variables of interest for the baseline model.

7.1 Steady State

Table 3: Simulation Results: Baseline Model Steady State

Equilibrium Type		Type 1 (misallocation)	
Aggregate Macro Variables		**Distribution of Entrepreneurs**	
Tax rate (connected firm)	21.27%	**% of Total Entrepreneurs**	
Tax rate (non-connected firm)	42%	**Connected**	
Total Output	2.07	Low skilled	49.5%
% of Total Output produced by connected	84%	High skilled	34.5%
% of Total Output produced by non-connected	16%	**Non-connected**	
Total Tax revenue	0.5	High skilled existing	3.6%
Tax/GDP	24%	High skilled new	12.4%
Gini Coefficient	0.5		
Total Welfare			
Elite	45.4		
Connected	70		
Non-connected	17		

Table 3 shows that Pakistan is in an equilibrium with misallocation in the steady state. The breakdown of the total output in the steady state shows that the share of the total output produced by the high skilled non-connected agents is significantly small at 16% compared to the 84% of the output produced by all the connected agents. Correspondingly, the share of total connected low skilled entrepreneurs is the highest at 49% and the share of non-connected entrepreneurs is the lowest equivalent to their share of the total output. It can be noticed that the fraction of existing non-connected entrepreneurs is relatively small only 3.6% of the total entrepreneurs. These entrepreneurs are the only non-connected entrepreneurs with positive net value gain of becoming entrepreneurs. There is high inequality between the tax rates charged on the connected entrepreneurs 21.27% and the non-connected 42%. The tax revenue to GDP ratio in this economy is 24%. The Gini Coefficient for the non Elite agents in the economy is found to be as 0.5. This Gini coefficient is computed using the share of total income or consumption given by the aggregate variable C in (27) for the fraction of connected and non-connected agent type in this economy. The Elite's probability to stay in power is quite high at 0.9. The total welfare is highest for the connected agents, all of whom are entrepreneurs in the steady state.

7.2 Transitional Dynamics

I consider a starting point where all agents are high skilled. As in Acemoglu (2008), this implies that in the initial period of the economy there will be some positive selection of only high skilled agents entering into entrepreneurship. Figure 5 show the computed value of M_t^H along the path, starting from $M_0^H = 1$, given the parameters values for σ^H and σ^L in Table 2, until it converges to its stationary value of $M^H = 0.41$. Notice that as I start at a point $M_0^H > M^H$, M_t^H is a decreasing sequence. Note that in figures 6-8 and figures 10-12, the green and black represents the transitional dynamics of the corresponding variables assuming always being in Type 1 and Type 2 respectively. Where as, the red line (dotted) represents the equilibrium path of the variables corresponding to the policy sequence $(\tau^c)^{t*}$ that maximises the period and lifetime welfare of the Elite as given by 28.

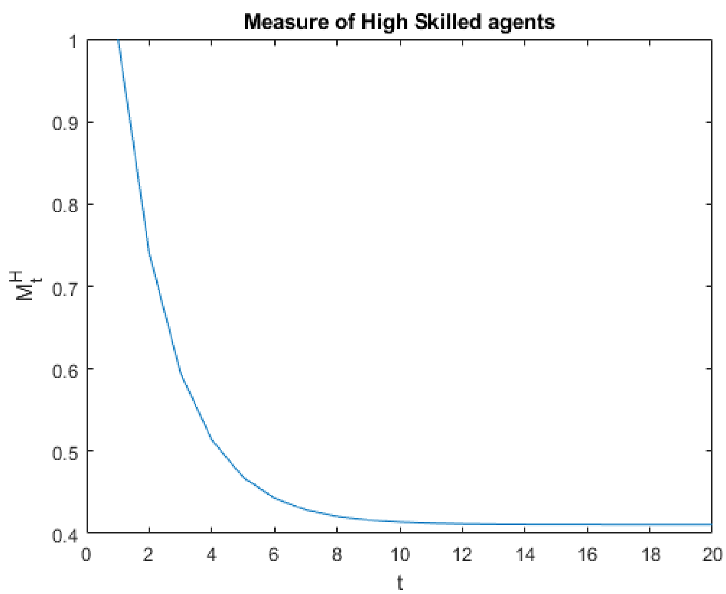


Figure 5: Measure of High skilled agents

Given M_t^H figure 6 show the optimal policy sequence $(\tau^c)^{t*}$. It can be seen that along the path the optimal $(\tau_t^c)^* = (\tau_t^c)^1$, so that every period the optimal $(\tau_t^c)^*$ that maximizes the Elite's period welfare given the corresponding equilibrium distribution of agents at period t is such that period equilibrium is always of Type 1 and $(\tau_t^c)^*$ is decreasing over time. Consequently, the Elite's ability to stay in power $\delta(\tau_t^c)$ is also increasing. Correspondingly, figure 8 show the period welfare of the Elite to be $(\tilde{u}_t)^* = (\tilde{u}_t)^1$ for all periods. Therefore, the lifetime welfare of the elite is maximized when the economy is always in a Type 1 equilibrium, along the equilibrium path. Figure 7 show that the output $(Y_t)^*$ is decreasing over time. This is due to the decrease in the measure of high skilled agents overtime and the preferential access to the market for the connected low skilled entrepreneurs compared to the non-connected high skilled entrepreneurs, owing to a $(\tau_t^c)^*$ below $\bar{\tau}$. Thus, in figure 7 when the measure of high skilled agents fall below 0.7 the equilibrium output decreases below the output of the economy if it was in a Type 2 equilibrium with no misallocation. Figure 9 show the fraction of existing non-connected entrepreneurs in equilibrium $(N_{Vt}^{ncHe})^*$ starting from $N_{V0}^{ncHe} = 0$ remains stationary at approximately 0.013.

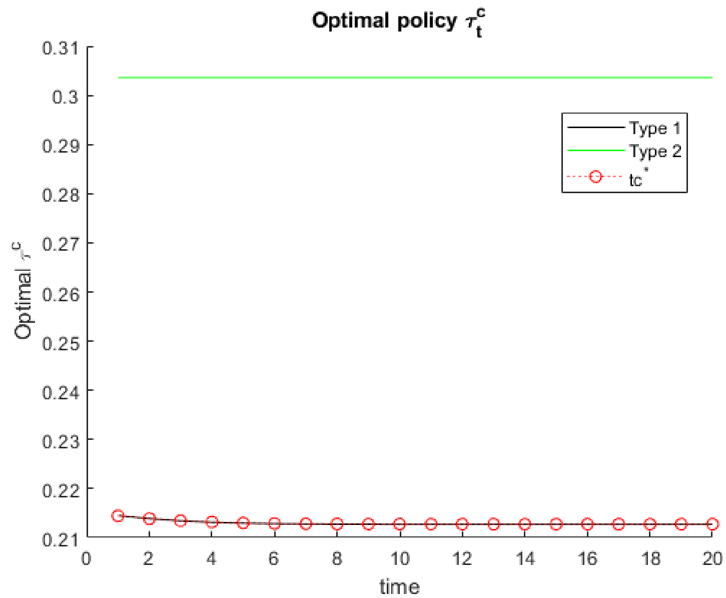


Figure 6: Optimal policy τ_t^c

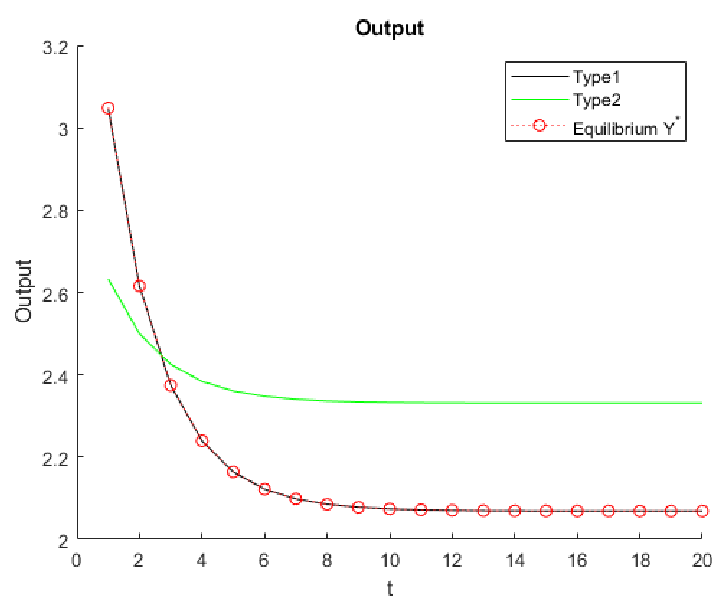


Figure 7: Output

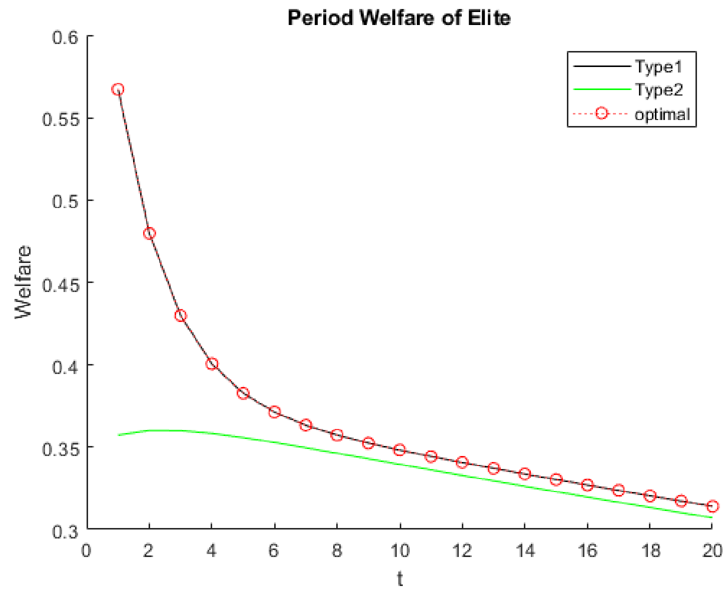


Figure 8: Period Welfare Elite

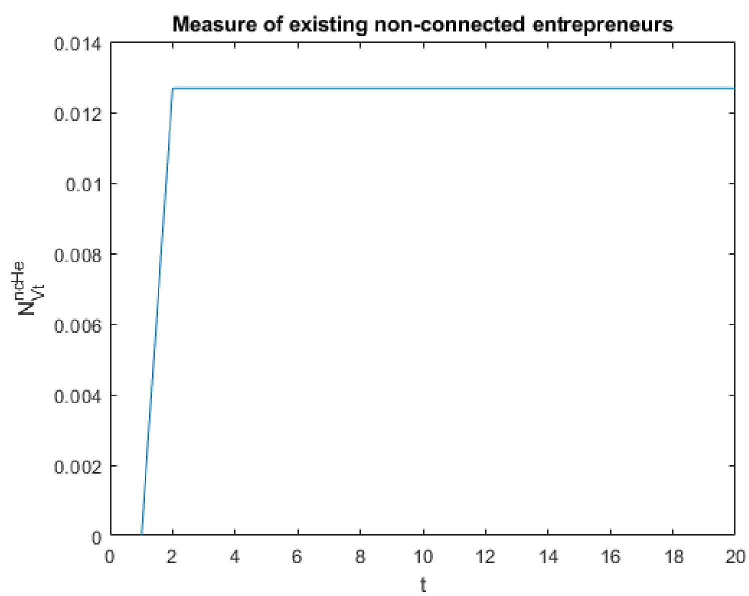


Figure 9: Measure of existing non-connected entrepreneurs

The next section discuss the impact on the key results for the baseline economy from carrying out counterfactual analysis in the absence of the external debt.

8 Counterfactual Exercises

I perform counterfactual analysis to determine whether the reduction of external flows in the economy causes a change in the Type of equilibrium observed in the simulated results. Subsection 8.1 presents the steady state results from the conterfactual exercise and is followed by subsection 8.2 which shows the impact of this exercise on the transitional dynamics including the policy and key variables of interest in the model.

8.1 Steady State

Table 4: Counterfactual exercise: Reduced External Flows Steady State

$D = 0.03$			
Equilibrium Type		**Type 2** (no misallocation)	
Aggregate Macro variables		**Distribution of Entrepreneurs**	
Tax rate on connected	30.3%	**% of Total Entrepreneurs**	
Δ Gain in Output	12%	**Connected**	
Δ Loss in the Output by connected	31%	Low skilled	0%
Δ Gain in the Output by non-connected	309%	High skilled	34.5%
Δ Gain in Tax revenues	74%	**Non-connected**	
Gain in Tax to GDP ratio	1.54 times	High skilled existing	14.5%
Gini Coefficient	0.38	High skilled new	51%
Change in Total Welfare			
ΔElite	-5.2%		
Δ Connected	-23%		
Δ Non-connected	32.4%		
Minimum reduction in the external flows to be in a steady state of Type 2	30%		

Table 4 shows the results of the baseline model by reducing the level of external debt $D = 0.03$ keeping the foreign aid F and other parameters fixed at the values of the baseline calibration. The simulation results show when the external debt is reduced by 30% economy is in Type 2 equilibrium with no misallocation. It is also note worthy to mention that in the steady state a minimum 30% decrease in the total external flows $(D + F)$ is enough for the economy to converge to a steady state of Type 2 equilibrium with higher total output.

Change in the aggregate macro variables show that the output of the economy increases and there is a significant gain of 12% compared to the benchmark model simulation. There is a substantial increase of 309% in the output produced by the connected high skilled agents and a corresponding decrease of only 49% in the output produced by the connected entrepreneurs. This is also reflected in the change of the composition of entrepreneurs. There are no low skilled entrepreneurs in the economy and the total share of non-connected entrepreneurs have increased to 65.5%. There is also a significant increase in the fraction of existing non-connected entrepreneurs. The existing non-connected entrepreneurs now consist of around 14.5% of the total entrepreneurs in the economy, a 11% increase from the baseline model. The probability of being removed from power increases by 5 times, but the welfare of the Elite is maximized from being in Type 2 equilibrium as there are higher revenues from taxes and entry barriers which can compensate for the loss in welfare from the decrease in the net external debt inflow. This reaffirms the presumptions that a decrease in external debt causes the Elite to be more dependent on the economy's own resources. To maintain it's revenues the Elite increase the tax rate on non-connected agents to $\tau^c = 0.30$. Thus, making it unprofitable for the low skilled connected to become or remain entrepreneurs. It is noteworthy to see that the model predicts a gain of about 1.54 times in the tax revenues to GDP ratio compared to the baseline results, with 60% decrease in the external flows. This is consistent with estimates from a prior study by Fenochietto & Pessino (2013), which estimate the maximum tax capacity of Pakistan to be approximately twice the actual tax to GDP ratio for 2018.

The share of the income is now more equally distributed as suggested by the lower Gini Coefficient compared to the baseline model. This is because now there is less favorable treatment towards the connected entrepreneurs in terms of lower taxes and there is more equal opportunity for the non-connected high skilled entrepreneurs to enter and produce.

The life time welfare of the non-connected agents increase by 47%. This is because there are now a higher percentage of existing non-connected entrepreneurs, who do not have to pay the entry costs, thus making higher profits. In comparison there is a decrease in the welfare of the Elite of 11%, due to a decrease in its consumption from lower flows and an increase in the δ due to higher τ_t^c. Similarly, there is also a decrease in the welfare of the connected of about 21.6% due to connected entrepreneurs paying higher taxes and the low skilled connected becoming workers.

8.2 Transitional dynamics

Figures 10 - 13 show the change in the equilibrium path of the key variables, starting with $M_0^H = 1$, when D is reduced to 0.03 from the initial period. Notice from figure 10 and 12, for the first seven periods the Elite maximizes its period welfare by selecting the optimal policy such that $(\tau_t^c)^* = (\tau_t^c)^1$ so that the economy remains in the Type 1 equilibrium. From period $t = 8$ onwards, the Elite's period welfare is maximized such that $(\tau_t^c)^* = (\tau_t^c)^2$ and the economy switches to equilibrium of Type 2 and remains in Type 2 equilibrium in the steady state. Figure 11 show that for the first two periods the economy remain in Type 1 equilibrium and does not lose the advantage of higher output due to higher proportion of connected entrepreneurs being high skilled and paying less taxes and investing higher capital compared to Type 2. In aggregate given the parameter values the economy stays below its maximum possible output for about fibe periods before transitioning and converging to an equilibrium Type 2 with higher output in the steady state. Figure 13 show that the measure of existing non-connected entrepreneurs remain low for the first five periods before sharply increasing when the economy switched to being in Type 2 and steadily increase until it converges to the steady state. Notice that it takes one extra period for the existing non-connected entrepreneurs to increase after the economy transitions to Type 2 equilibrium. This is because the measure of existing non-connected entrepreneurs depends on the measure of total non-connected entrepreneurs in the previous period.

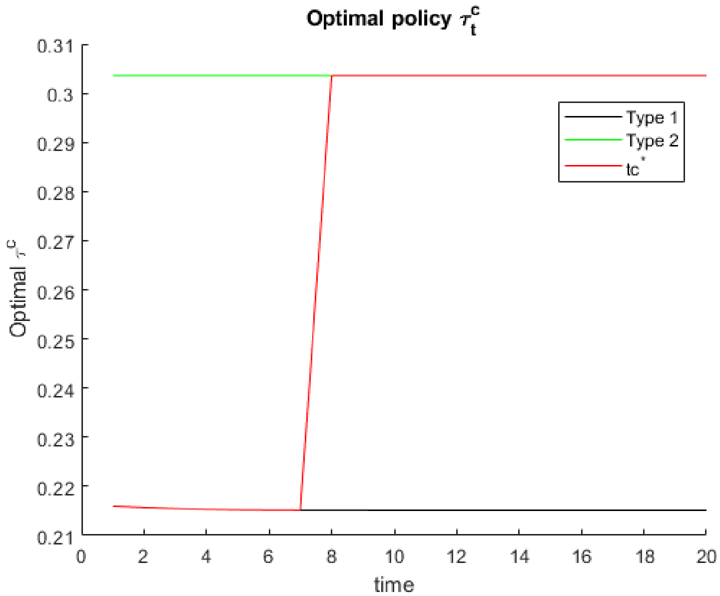


Figure 10: Optimal policy τ_t^c

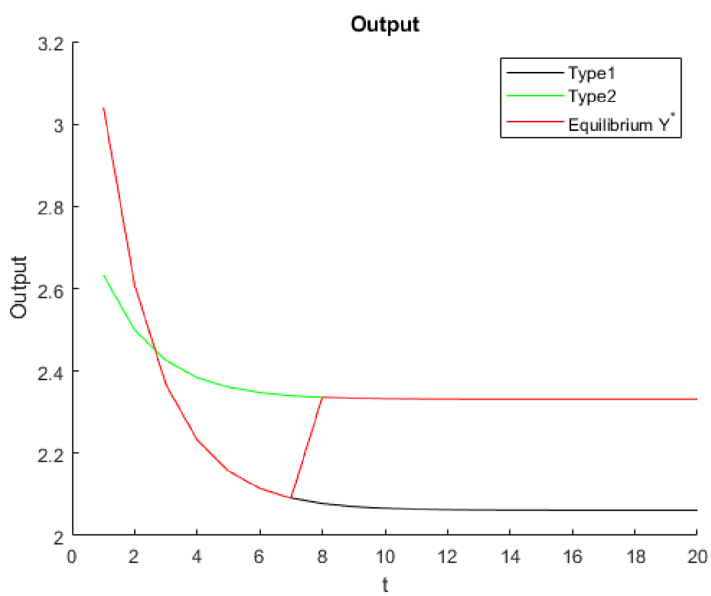


Figure 11: Output

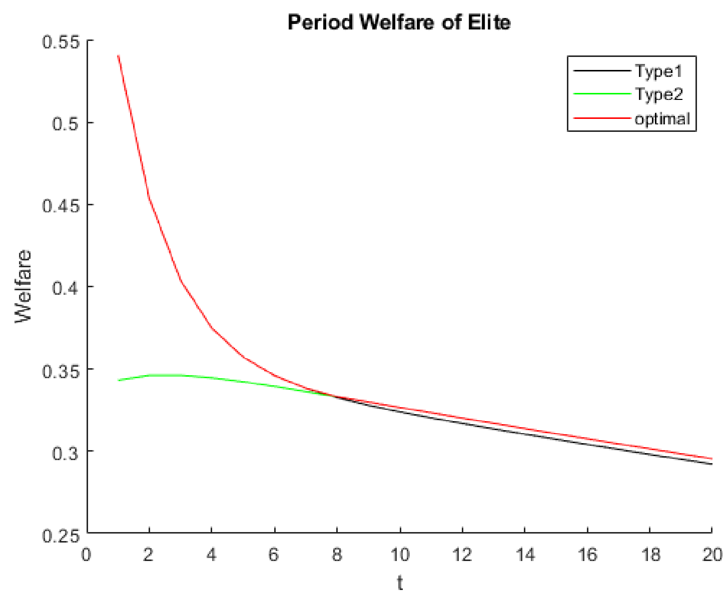


Figure 12: Period Welfare Elite

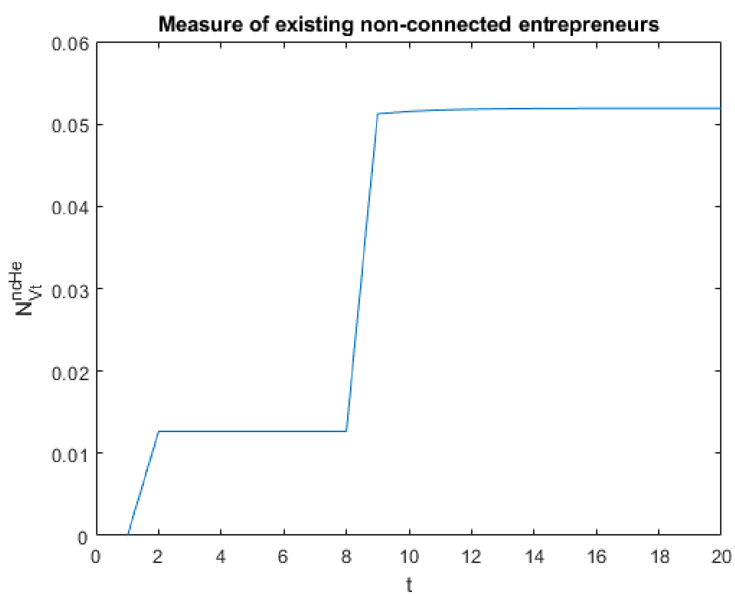


Figure 13: Measure of existing non-connected entrepreneurs

Notice that any decrease in the external flows greater than 30% will increase the welfare of the non connected high skilled entrepreneurs by a higher percentage. This is because lower the non-tax revenues for the Elite, the quicker will be the transition to an equilibrium with no misallocation (Type 2). Correspondingly, the output and the measure of existing non connected entrepreneurs earning positive profits will also increase sooner contributing to the increase in their welfare. Section A.3 in the Appendix show the results of simulations with D=0 corresponding to a 60% decrease in the external flows.

8.3 Sensitivity Analysis

I perform a sensitivity analysis by changing some of the key calibrated parameters to see the impact on the results of the benchmark model steady state reported in Table 3. The results of the sensitivity analysis are summarized in Table 5. It can be observed that the main results of the baseline model do not change much with respect to small changes in the parameter values of $\bar{L}$,a, ϕ^c. However, the results of the model are sensitive to the changes in the values of the entry barriers b^n. Lowering entry barriers per worker below 0.261 would result in the baseline economy being already in Type 2 equilibrium keeping the other parameters fixed. This is an expected result of the model as we start with the premise of an economy with high entry barriers.

9 Discussion: Policy Recommendations

The theoretical and quantitative analysis in this paper pave the way for a discussion surrounding conditions attached with external debt and aid. Specifically, external flows without conditions may not be the best way to reduce misallocation in developing countries with corrupt politicians reliant on patronage systems. The baseline model simulation results suggest that if the external aid and external debt programs by the donor entities are structured without appropriate conditions or no strings attached, this allows the government to maintain situations of high misallocation in these economies. This includes fiscal policy encouraging

Table 5: Sensitivity Analysis: Steady State

Baseline model	Type 1	τ^c =21.71%	
Parameters	**Type**	$\boldsymbol{\tau^c}$	**Output gain in Type 2**
$\bar{L}$			
2.5	Type 1	20.39%	8.4%
2.8	Type 1	21.71	12% (Baseline model)
3.0	Type 1	22.43%	12.5%
b^n			
0.29	Type 1	20.2%	11%
0.27	Type1	21.27	12% (Baseline model)
0.26	Type 2	29.6%	None
$\delta(\tau^c)$			
a			
$a = 20$	Type 1	19.82%	7.27%
a=25	Type 1	21.27	12% (Baseline model)
$a = 30$	Type 1	23.82%	13.2%
ϕ^c			
.28	Type1	20.58%	9.63%
.30	Type 1	21.27	12% (Baseline model)
.32	Type 1	22.46%	13.35%

the low productivity entrepreneurs to enter and produce, and a lack of incentive to reform inefficient institutions that cause high barriers to entry into entrepreneurship. Based on the simulation and comparative analysis results in section 7 and 8, there are three policy recommendations. In particular, the following conditions if included and enforced as part of the aid and debt packages can assist with removing this type of misallocation in developing economies.

Firstly, if the debt and aid is provided without any conditions, the levels of flows given can be reduced to the extent that this encourages the government to be depended on their own resources to maintain their fiscal budget. For the case of Pakistan the model shows that a 30% reduction in the external debt can bring about an improvement in the total output of the economy. Secondly, condition can be place on the minimum level of direct tax revenues that should be raised, for these economies to be eligible for these flows. In the model for Pakistan, an increase in the tax revenue of 66% of their current total direct taxes in the steady state would help the economy to be in a stationary equilibrium with no misallocation. This would increase Pakistan's total output in the steady state keeping the current level of external flows. Interestingly the transitional dynamics given in figure 14 show that the condition imposed on the minimum level of tax revenues to be raised by the Elite cannot be satisfied even with the maximum tax of $\tau_t^c = \alpha$ on the connected $\forall\ t$ except for $t = 1$. Thus, Type 1 equilibrium will not exist for any $t > 1$. Notice that in this model there are only direct corporate taxes, however if the taxes were on the income the results would be similar. An increase in taxes which are lump sum to pay the debt will not have any impact on the type of equilibrium in this model. This could be synonymous to an increase in fees charged by the government for example for vehicle registration, license fee, electricity connection etc.

Lastly, most important recommendation would be to place conditions to utilize external debt and aid to implement reforms that lower the existing entry barriers, for the new firms entering the industry. This could result in these external flows in improving the economic growth, inequality and the misallocation in the developing economies. In our baseline model as shown in figure 15, a decrease of just 4% of the current value of the entry barriers per worker for a firm would result in Pakistan's economy converging to a Type 2 equilibrium with no misallocation in the steady state.

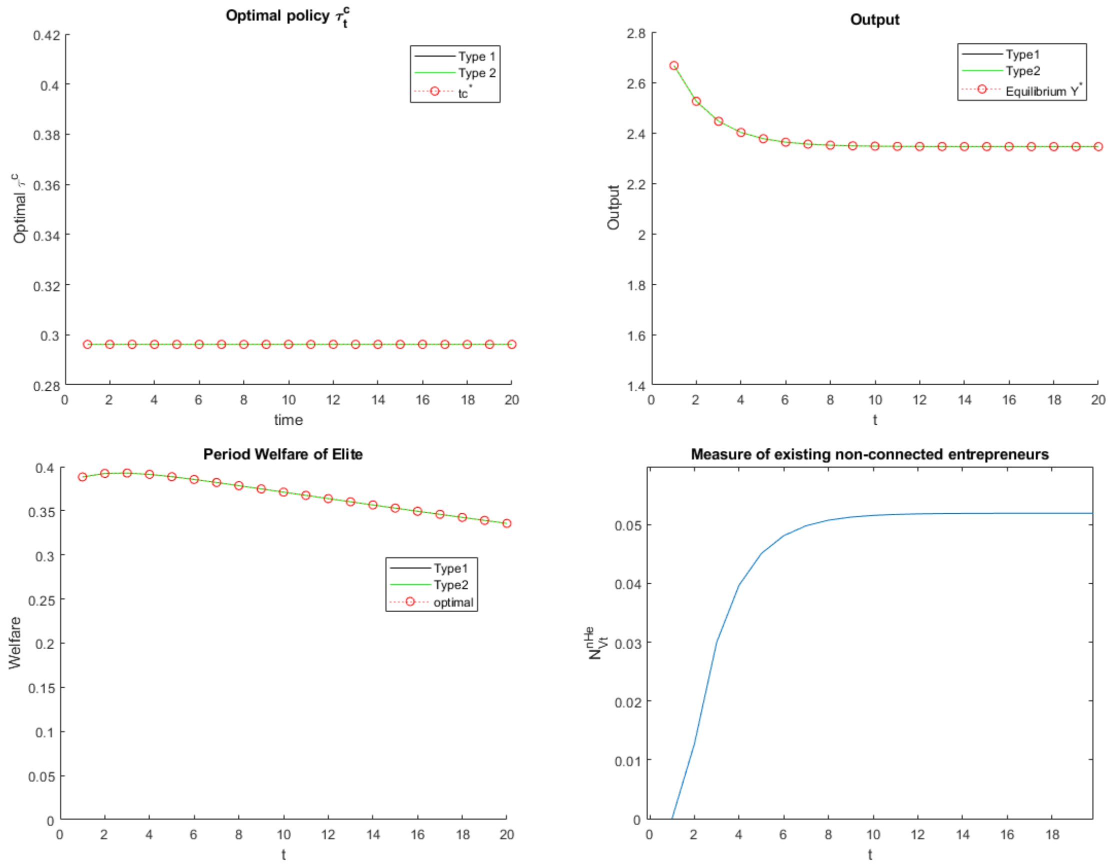


Figure 14: Equilibrium Path with a constraint on the minimum level of Tax revenues

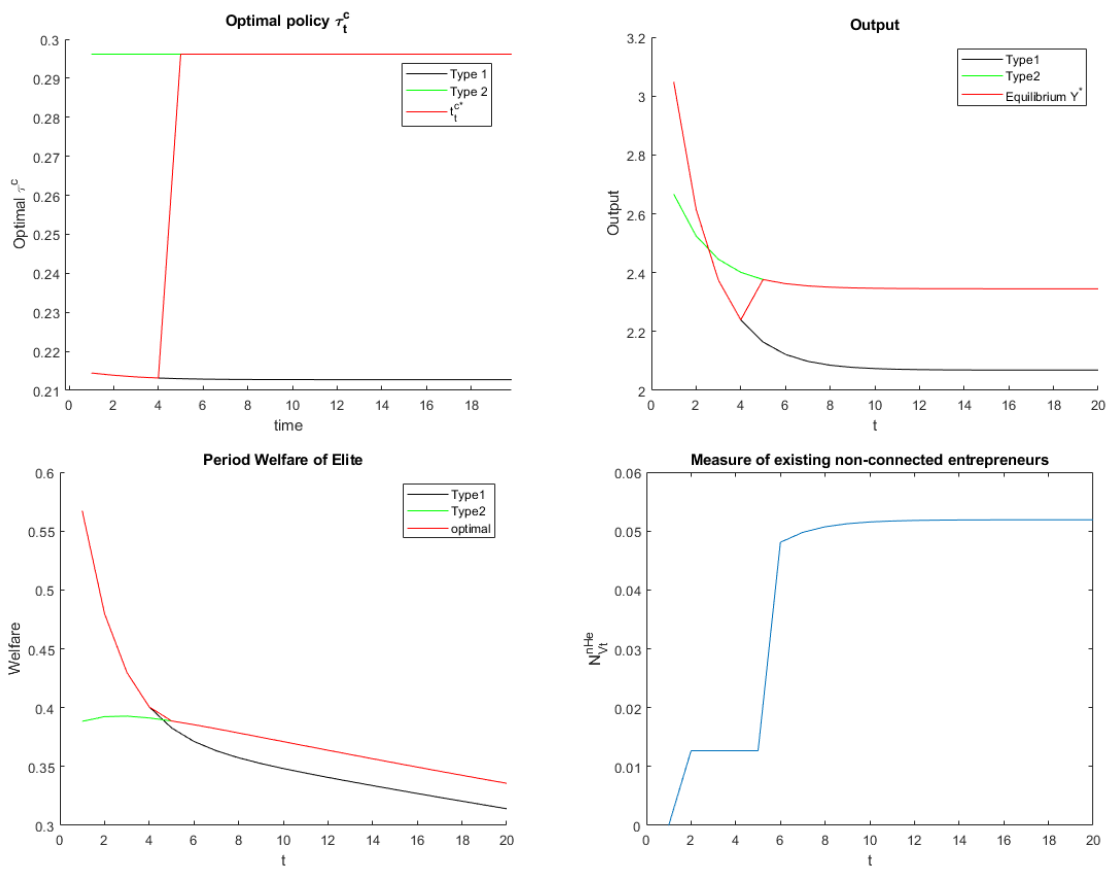


Figure 15: Equilibrium Path with lower entry barrier B^n

10 Conclusion

In this paper I document a mechanism that shows how unconditional external debt and foreign aid can be counterproductive in reducing misallocation and promoting growth in some developing countries. Using Pakistan as an example I show both empirically and theoretically, that in the presence of a system of political patronage high external flows perpetuate low growth and high inequality. I first find empirical evidence that listed Pakistan politically connected firms pay lower effective tax rates and that this preferential treatment increases significantly with the public external debt to GDP ratio. I then develop a political economy model that proposes a mechanism that rationalizes these findings. In the model, heterogeneous entrepreneurs connected with the government receive lower effective tax rates in return for political support to corroborate the above findings. The model is able generate different levels of misallocation depending on whether low skilled connected individuals become entrepreneurs. The main result of the model is that, under certain conditions, the level of external flows determines whether the economy is in an equilibrium with or without resource misallocation. Although the empirical methodology does not directly derive from the model, both advocate similar results. The main finding of this paper is that the calibrated model is able to generate a reduction in tax differential and consequently an increase in Pakistan's output when unconditional external flows are reduced.

The significance of this study lies in understanding and quantifying the existence of the preferential treatment received by politically connected firms and the unintended effects of external flows on the intensity of this preferential treatment. Therefore, this study speaks to the effective structuring of aid and debt programs for international donor organizations. In particular, the model indicates that if these flows are provided conditional on the recipient nation's commitment to either increase their direct tax revenues or reduce entry barriers for firms, these would eliminate resource misallocation in their economy. Even though this paper provides a framework of how low tax rates for the connected firms lowers output when external flows are high, our understanding of the effect of the different types of preferential treatments on economic growth is still at an early stage. Thus, the results from this paper can be used in future research in the following ways. First, to study the effects of other forms of preferential treatment to the connected on the economic growth. Second, to address the question of the optimal level of external flows that would increase economic growth of developing economies in the presence of these treatments.

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A Appendix

A.1 Further details of the empirical analysis

Table 6 presents the final distribution of the sample firms based on connection. It shows that given the definition of politically connectedness for the sample with 268 firms 63.4% of the firms are identified as politically connected for the year 2013-2017 and 63.8% of the firms are identified as politically connected for the years 2018-2019. The distribution of connected firms is quite similar excluding the firms with extrapolated values. This is suggestive of the fact that there a significant proportion of firms in Pakistan have directors associated with politics and it is consistent over the years. Further more, the table shows that 18 additional firms become politically connected after the general election of 2018 and 17 firms lost their political connections as none of their key board of directors took part in the 2018 general elections for both the samples.

Table 6: Sample Distribution of firms based on Connections

Total number of firms with available data	261	
Years (Period)	**2013-2017**	**2018-2019**
No. Connected firms per year	168	169
No. non-connected firms per year	93	92
% of connected firms per year	**64.4%**	**64.8%**
Total observations per year	261	261
Total observations for 2013-2019	1827	

Total number of firms with available and imputed data	268	
Years (Period)	**2013-2017**	**2018-2019**
No. Connected firms per year	170	171
No. non-connected firms per year	98	97
% of connected firms per year	**63.4%**	**63.8%**
Total observations per year	268	268
Total observations for 2013-2019	1876	
Firms with changed connectivity status for 2018-2019		
No of firms with POLCON 0 to 1	18	
No of firms with POLCON 1 to 0	17	

Table 7 presents the summary statistics for the final dependent and the explanatory variables that are used in the analysis for the two samples containing 261 firms and the 268 firms. The Table shows that the mean ETR for the final cleaned and recoded variables in the sample is around 20% for all the firms. The mean EDGDP is 29.5% for Pakistan for the period of 2013-2019, with the mean public external debt to GDP ratio being 25.2%. The external debt to GDP ratio for Pakistan which comprises of more than 80% of the county's total external debt fluctuates between 23.8% and 43.6% during the sample period.

Following the empirical studies involving external debt, I construct two ratios (a) Public and publicly guaranteed external debt to GDP ratio (EDPGDP) and (b) Total external debt to GDP ratio (EDGDP). The results show that indeed the preferential treatment effect is less strong when using (EDGDP). Table 8 presents the additional analysis for 261 firms in columns (1) and (5) for specification 1 and 2. Column (3) and (4) represents the results for the two sample with 261 and 268 firms using the total external debt to GDP ratio (EDGDP)

Table 7: Descriptive statistics 1

Variables	**mean**		**sd**		**min**		**max**	
No of firms	261	268	261	268	261	268	261	268
ETR	0.207	0.204	0.301	0.300	0	0	1	1
POLCON	0.645	0.635	0.479	0.481	0	0	1	1
EDPGDP	0.252	0.252	0.0447	0.0447	0.215	0.215	0.347	0.347
EDGDP	0.295	0.295	0.0637	0.0637	0.238	0.238	0.436	0.436
Control Variables								
SIZE	1.126	1.136	0.827	0.826	-1.948	-1.948	3.366	3.366
COLLATERAL	0.556	0.543	0.227	0.238	-0.0409	-0.0409	1	1
ROA	0.0439	0.0442	0.134	0.133	-2.792	-2.792	0.421	0.421
GOVGDP	0.113	0.113	0.00345	0.00345	0.108	0.108	0.117	0.117
Lending(%)	0.102	0.102	0.0163	0.0163	0.0821	0.0821	0.1272	0.1272
FX	0.00905	0.00905	0.00111	0.00111	0.00667	0.0066	0.00989	0.00989

Note: ETR = (Tax expenses- Deferred tax expenses)/(Earnings before interest rate and tax); POLCON=1 if the firm has a board of director who is politically connected; 0 otherwise; EDPGDP = (public and publicly guaranteed external long term debt stock+ short term external debt stock in US dollars)/ (Nominal GDP in US dollars) ; EDGDP = (Total external debt stock in US dollars)/(Nominal GDP in US dollars); SIZE= Log of Total Assets; COLLATERAL= (Total Assets- Total current Assets)/(Total Assets); ROA= (Earnings before interest and tax)/(Total Assets); GOVGDP= (Total government expenditure in US dollars)/(Nominal GDP in US dollars); Lending(%)= Annual average SBP lending interest rate; FX= Average annual foreign exchange rate pf Pakistani Rupee in terms of US dollars.

for the following specification:

$$ETR_{it} = \alpha_0 + \beta_1 \cdot POLCON_{it} + \beta_2 \cdot EDGDP_t + \beta_3 \cdot POLCON_{it} * EDGDP_t$$
$$+\lambda_1 \cdot X_{it} + \lambda_2 \cdot X_t + \psi_i + \epsilon_{it}$$

It can be noticed that the coefficients for $POLCON$, $POLCON*EDPGDP$ and $POLCON*EDGDP$ are not significantly different between the two samples. Also note that for the specification (2) using public and publicly guaranteed external debt, the coefficient for $POLCON * EDPGDP$ is significantly higher than of the specification using the total external debt stock $POLCON * EDGDP$. This supports the argument that an increase in the external debt provided to the government leads to a greater increase in the preferential treatment for the politically connected firms in terms of lower effective tax rates.

A.2 Economic Equilibrium Proofs

Proof. Given $\tau_t^n = \alpha$ and lemma 5.6 the equilibrium wage $w_t^* = w_t^{nHr}$ for cases 1, 2 and 3, for which the $NG_t^{nHr} = 0$ and $V_t^{nH} - W_t^{nH} - b^n \bar{L} = 0$, $\forall t$. Similarly, if cases 3 and 4 were

Table 8: Regression table

	(1)	(2)	(3)	(4)	(5)	(6)
	ETR	ETR	ETR	ETR	ETR	ETR
No of firms	261	268	261	268	261	268
POLCON	-0.0832**	-0.0837**	0.135*	0.127*	0.175**	0.168**
	(0.0341)	(0.0339)	(0.0749)	(0.0734)	(0.0876)	(0.0857)
EDGDP			-0.442			
			(0.839)			
POLCON*EDGDP			-0.681***	-0.656***		
			(0.208)	(0.203)		
EDPGDP					0.0359	0.0858
					(0.761)	(0.747)
POLCON*EDPGDP					-0.953***	-0.926***
					(0.298)	(0.290)
SIZE	-0.0216	-0.00458	-0.0269	-0.0259	-0.0283	-0.0273
	(0.0416)	(0.0390)	(0.0414)	(0.0405)	(0.0414)	(0.0406)
COLLATERAL	-0.166**	-0.150**	-0.163**	-0.159**	-0.163**	-0.159**
	(0.0646)	(0.0656)	(0.0637)	(0.0638)	(0.0643)	(0.0639)
ROA	-0.283***	-0.293***	-0.276***	-0.272***	-0.274***	-0.270***
	(0.0663)	(0.0651)	(0.0661)	(0.0656)	(0.0661)	(0.0657)
GOVGDP			-0.141	-0.781	-4.807	-4.885
			(8.646)	(8.503)	(6.713)	(6.600)
LENDING(%)			-0.318	-0.458	-1.164	-1.200
			(1.168)	(1.148)	(0.821)	(0.807)
FX			0.00292	0.00271	0.00212	0.00199
			(0.00234)	(0.00230)	(0.00208)	(0.00205)
Firm fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Time fixed effects	Yes	Yes	No	No	No	No
No of Observations	1827	1876	1827	1876	1827	1876

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Variable definitions: ETR = (Tax expenses- Deferred tax expenses)/(Earnings before interest rate and tax); POLCON=1 if the firm has a board of director who is politically connected; 0 otherwise; EDPGDP = (public and publicly guaranteed external long term debt stock+ short term external debt stock in US dollars)/ (Nominal GDP in US dollars) ; EDGDP = (Total external debt stock in US dollars)/(Nominal GDP in US dollars); SIZE= Log of Total Assets; COLLATERAL= (Total Assets-Total current Assets)/(Total Assets); ROA= (Earnings before interest and tax)/(Total Assets); GOVGDP= (Total government expenditure in US dollars)/(Nominal GDP in US dollars); Lending(%)= Annual average SBP lending interest rate; FX= Average annual foreign exchange rate of Pakistani Rupee in terms of US dollars.

allowed, given the assumption 1 and that every period θ fraction of existing entrepreneurs die. For, cases $w_t^{nHr} \geq w_t^{nLe}$ or $w_t^{nHr} < w_t^{nLe}$, $w_t^* = w_t^{nHr}$ as some non-connected new high skilled entrepreneurs will always be required to enter the equilibrium as the total number of existing entrepreneurs will be less than the total firms $\frac{1}{L}$ required for market clearing in equilibrium. I have shown that $w_t^* = w_t^{nHr}$ for cases 1, 2, 3, 4 and 5 $\forall t$, given $\tau_t^n = \alpha$, I can write $V_t^{nj} = V_{t+1}^{nj} = V^{nj}$. Therefore if $w_t^{nHr} > w_t^{nLe}$ then $V^{nL} - W^{nL} < V^{nH} - W^{nH} - b^n \bar{L} = 0$. Where $V^{nL} - W^{nL} = \frac{\alpha}{1-\alpha}(1-\alpha)^{1/\alpha}(A^L - A^H) + b^n - \beta(1-\theta)\sigma^H b^n < 0$. Thus, it follows that if $\frac{\frac{\alpha}{1-\alpha}(1-\alpha)^{1/\alpha}(A^H - A^L)}{1-\beta(1-\theta)\sigma^H} > b^n$, then $w_t^{nHr} > w_t^{nLe}$. □

A.3 Additional Simulations

Table 9: Welfare Analysis Reduced External flows D=0

Equilibrium Type $D = 0$	**Type 2** (no misallocation)
Change in Total Welfare	
ΔElite	-11%
Δ Connected	-21.6%
Δ Non-connected	47%

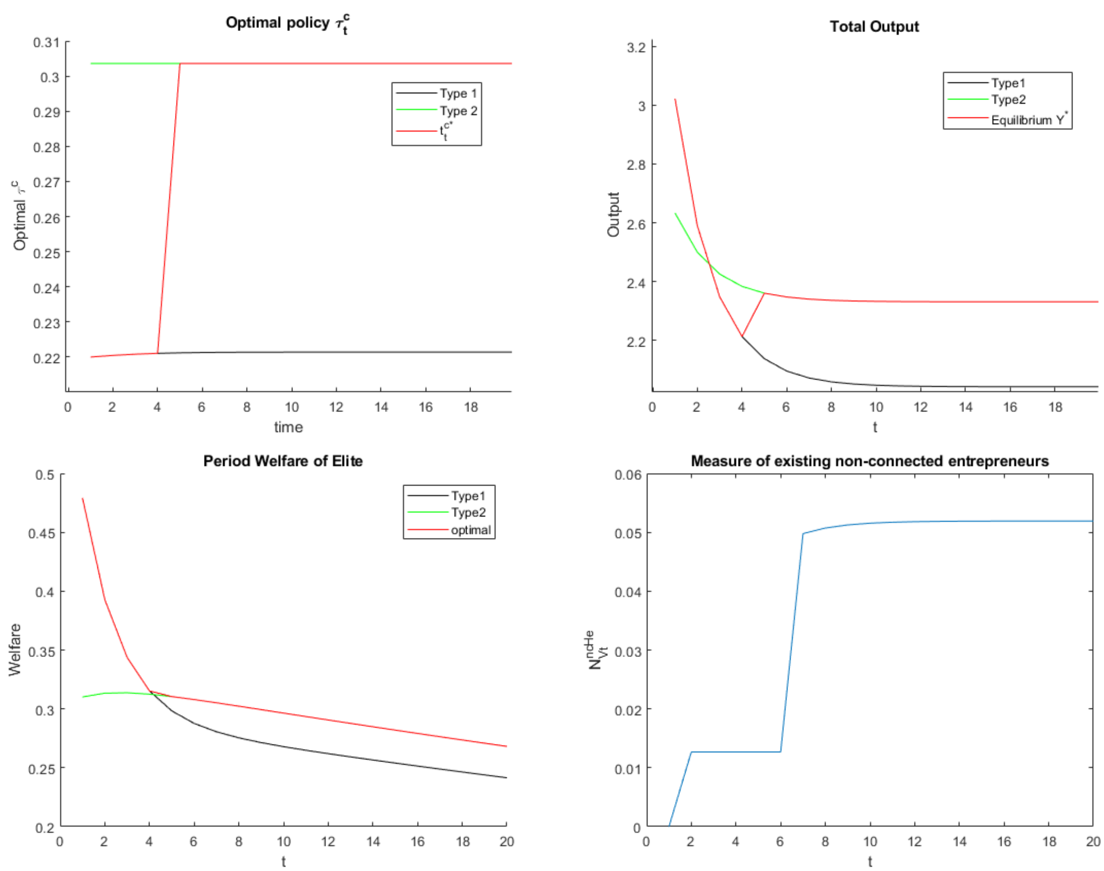

Figure 16: Equilibrium Path with D=0 (60% reduction in External Flows)